

# **ANALYSIS OF STRATIFIED AND CLOSELY SPACED JETS EXHAUSTING INTO A CROSSFLOW**

*by H. Ziegler and P.T. Wooler*

*Prepared by*

**Northrop Corporation, Aircraft Division  
Hawthorne, California**

*for Langley Research Center*

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**H. Ziegler  
P.T. Wooler**

## FOREWORD

This report documents the results of an effort to develop analytical methods for determining flow fields due to stratified and closely spaced jets exhausting into a cross-flow. In conjunction with this study, a wind tunnel test program was conducted to generate data against which analytical results could be compared.

Mr. M. F. Schwendemann directed the experimental phase of this investigation, which is documented in Northrop report NOR 73-98.

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# ANALYSIS OF STRATIFIED AND CLOSELY SPACED JETS EXHAUSTING INTO A CROSSFLOW

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H. Ziegler and P. T. Wooler  
Northrop Corporation, Hawthorne, California

## SUMMARY

Procedures have been developed for determining the flow field about jets with velocity stratification exhausting into a crossflow. Jets with three different types of exit velocity stratification have been considered, namely

- Jets with a relatively high velocity core
- Jets with a relatively low velocity core
- Jets originating from a vaned nozzle

The procedure developed for a jet originating from a high velocity core nozzle is to construct an equivalent nozzle having the same mass flow and thrust but having a uniform exit velocity profile. Calculations of the jet centerline and induced surface static pressures have been shown to be in good agreement with test data for a high velocity core nozzle.

The equivalent ideal nozzle has also been shown to be a good representation for jets with a relatively low velocity core and for jets originating from a vaned nozzle in evaluating jet-induced flow fields.

For the singular case of a low velocity core nozzle, namely a nozzle with a dead air core, and for the vaned nozzle, an alternative procedure has been developed. The internal mixing which takes place in the jet core has been properly accounted for in the equations of motion governing the jet development. Calculations of jet centerlines and induced surface static pressures show good agreement with test data for these nozzles.

A method for treating two-jet configurations, formulated in an earlier investigation, has been extended to include mutual interference effects between the two jets in addition to the jet blockage effects already considered. Comparisons are made between calculations and test data for a number of jet configurations.

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## INTRODUCTION

A fundamental problem in the development of methods for predicting aerodynamic characteristics of lift-jet, vectored thrust and lift-fan V/STOL aircraft is that of formulating a mathematical model to estimate the effects of the propulsion system efflux interaction with a crossflow. During the transition flight phase, this efflux is directed at large angles to the freestream and has a significant influence on the aircraft aerodynamics as well as on the stability and control requirements. Consequently, a considerable amount of research activity, both experimental and analytical, has been devoted to the development of an understanding of this flow problem and also to the development of methods to calculate the resulting interference flow fields.

A number of analytical formulations of the problem of a single jet exhausting into a crossflow exist, and details of the different approaches may be found in reference 1. An approach to the problem of a single, normally exhausting jet, which appeared to offer possibilities of treating more complex flow configurations, may be found in reference 2. An entrainment model was developed from dimensional analysis and physical considerations. The force on the jet boundary, as a result of the pressure differential around the jet, was accounted for by a crossflow drag. The geometry of the jet cross section was represented by an ellipse and the continuity and momentum equations were solved to provide the jet path. The jet-induced velocity field was then determined by replacing the jet by a distribution of sinks and doublets. Using an image system for the flat plate and lifting surface theory for the finite wing, it was then possible to determine the jet-induced pressure distribution on these two types of surfaces.

Induced surface static pressure distributions around single jets exhausting normally into a crossflow have been determined in references 3, 4, 5 and 6. Jet decay investigations for single jets exhausting at  $90^\circ$  into a crossflow were conducted in references 7 and 8. Jet centerlines (usually defined as the position of maximum total head in the jet) have been obtained for single jets exhausting at various angles into the crossflow (references 9, 10, and 11).

The analytical model described above has been further extended in reference 12 to treat jets exhausting into arbitrarily directed crossflows as well as multiple-jet

configurations. In the case of multiple-jet configurations it was assumed that the leading jet (jet farthest upstream) develops independently of the downstream jets until intersection occurs. It was also assumed that downstream jets behave as single jets developing into a crossflow of reduced dynamic pressure. An arbitrary jet configuration could then be treated as a combination of discrete jets, with the induced velocity components due to each jet being additive at each control point. Data from the wind tunnel investigation of reference 13 were used to substantiate the assumptions made in the development of the analytical model. Empirical relationships postulated in the development of the model were established quantitatively.

In general, the exit velocity profile of a V/STOL fan or jet will not be uniform (see reference 14, for example). Variations in dynamic pressure decay may also exist. In the hover case, it has been shown (reference 15) that jets with different decay rates produce different induced aerodynamic forces, so that similar effects are expected with the jet exhausting into a crossflow.

It has been the purpose of this study to develop methods for determining flow fields due to stratified and closely spaced jets exhausting into a crossflow. In conjunction with this study a wind tunnel test program has been conducted to generate data against which analytical results can be compared.

## SYMBOLS

$a$	ratio of inner to outer diameter for an annular nozzle (see sketch 2)
$A$	exit area of an equivalent ideal nozzle
$A_j$	jet cross-sectional area
$b$	core mixing parameter, $ad_0/\ell$
$C$	circumference of jet cross section
$C_D$	crossflow drag coefficient of jet cross section
$C_p$	pressure coefficient, $(p - p_\infty)/q_\infty$
$d$	length of major axis in elliptical representation of jet cross section
$d_0$	jet exit diameter
$d^*$	$d/d_0$
$d_e$	effective jet exit diameter, obtained from equivalent ideal nozzle considerations (see equation (11))
$d_2$	diameter of downstream jet (see sketch 4)
$E$	entrainment crossflow per unit length of jet (see equation (1))
$E_1, E_2, E_3$	entrainment parameters (see equation (1))
$F$	nozzle thrust
$K$	jet blockage factor, $(d_0 - \Delta)/d_0$
$\ell$	length of core region for an annular nozzle (see sketch 2)
$m$	jet exit velocity parameter, inverse velocity ratio $U_{j0}/U_\infty$
$m_{i\text{eff}}$	inverse velocity ratio for $i^{\text{th}}$ segment of jet in mutual interference computations
$\dot{m}$	jet mass flow
$\dot{m}_0$	jet mass flow at exit
$M$	Mach number
$p$	static pressure
$p_\infty$	crossflow static pressure
$q$	jet dynamic pressure at exit of an ideal nozzle
$q_\infty$	crossflow dynamic pressure, $\frac{1}{2}\rho U_\infty^2$
$q_e$	effective crossflow dynamic pressure for downstream jet when jets are aligned in freestream direction
$q'_e$	$q_e$ when leading jet does not exhaust normally into the crossflow (see equation 22)



# SYMBOLS (Continued)

$q_{\beta}$	effective crossflow dynamic pressure for downstream jet when jets are not aligned in the crossflow direction
$q_j, q_{j0}$	jet dynamic pressure, jet dynamic pressure at exit
$q_{je}$	effective jet dynamic pressure at exit
$R$	gas constant
$s$	spacing between two jets
$T_o$	stagnation temperature
$U_{\infty}$	crossflow speed
$U_j, U_{j0}$	jet speed, jet speed at exit
$U_j^*$	nondimensionalized jet speed, $U_j/U_{j0}$
$U_{\infty}/U_{j0}$	velocity ratio, $[q_{\infty}/q_{j0}]^{1/2}$
$\bar{u}$	induced velocity vector for freestream modification (see sketch 5)
$X, Y, Z$	Cartesian coordinate system
$X^*, Z^*$	distances in and normal to the crossflow direction normalized by $d_o$
$\beta$	sideslip angle
$\gamma$	ratio of specific heats
$\Delta$	overlap of downstream jet by upstream jet
$\delta_j$	jet deflection angle at exit (see sketch 8)
$\rho$	density
$\theta$	angle between local velocity vector and the normal to the crossflow vector (see sketch 1)
$\theta_o$	value of $\theta$ at jet exit
Subscripts	
$i$	value in $i^{th}$ segment of jet in mutual interference computations
$o_i$	initial value for $i^{th}$ segment of jet in mutual interference computations

## VELOCITY STRATIFICATION EFFECTS

In general, the exit velocity profile of a V/STOL fan or jet engine will not be uniform. Variations in dynamic pressure decay may also exist. The jet exit velocity profile and variations in dynamic pressure decay have an effect on the jet-induced velocity field in a crossflow and consequently influence the aircraft aerodynamics.

In this report three different types of nozzle flows are considered, namely

1. Annular nozzle with high velocity core
2. Annular nozzle with low velocity core
3. Vaned nozzle

The annular nozzle with high velocity core is representative of the exhaust flow from a turbofan engine. The annular nozzle with low velocity core models the flow from a lift-fan engine. The vaned nozzle is sometimes considered for vectored thrust concepts.

In earlier investigations, references 2 and 12, a mathematical model was developed for the flow about a subsonic turbulent jet exhausting at an angle into a uniform crossflow. In these studies the jet was assumed to be deflected in the crossflow direction due to entrainment of crossflow fluid and also due to jet blockage.

Consider a circular jet exhausting at a right angle into a uniform mainstream, as shown in figure 1. The entrainment of crossflow fluid was represented, in reference 2, by the expression

$$\frac{E}{\rho U_{\infty} d_o} = E_1 d^* \cos \theta + \frac{E_2 (m U_j^* - \sin \theta) C / d_o}{1 + E_3 \cos \theta / m U_j^*} \quad (1)$$

The entrainment parameter  $E_2$  is obtained from static results for the jet and may vary from nozzle to nozzle, whereas  $E_1$  and  $E_3$  have been determined empirically and are independent of the nozzle configuration.

The equations of motion for the jet are, according to reference 2,

$$\cos \theta \frac{d}{dZ^*} \left( \frac{A_j}{d_o^2} m U_j^* \right) = \frac{E}{\rho U_\infty d_o} \quad (2)$$

$$\cos \theta \frac{d}{dZ^*} \left( \frac{A_j}{d_o^2} m^2 U_j^{*2} \right) = \frac{E}{\rho U_\infty d_o} \sin \theta \quad (3)$$

$$\frac{A_j}{d_o^2} \frac{m^2 U_j^{*2} X^{*''}}{[1+(X^{*'})^2]^{3/2}} = \frac{E}{\rho U_\infty d_o} \cos \theta + \frac{C_D}{2} \cos^2 \theta d^* \quad (4)$$

where primes denote differentiation with respect to  $Z^*$ .

From equations (1) through (4) it is observed that  $m$  is a parameter and that  $d_o$  is the length dimension for normalizing purposes. In the case of a standard convergent subsonic nozzle,  $m$  and  $d_o$  are clearly defined as the square root of the ratio of the jet exit to mainstream dynamic pressures and the nozzle exit diameter, respectively. One of the approaches to treating jets with stratified exit flow characteristics explored as part of this study is to employ the analytical model of reference 2, contingent upon an appropriate determination of  $m$  and  $d_o$  for the nozzle.

In order that the analytical methods developed in this study may be used with confidence to calculate induced flow fields, pressure distributions and forces and moments on adjacent aerodynamic surfaces, it is desirable to make comparisons between calculations and test data for induced surface pressures on a simple geometric shape due to stratified jets exhausting into a crossflow.

To generate test data against which analytical results can be compared, a wind tunnel test program has been conducted and documented in detail in reference 16. A four-foot diameter circular plate containing the nozzles was aligned with the tunnel flow. The plate contained pressure taps to determine the surface static pressure distributions around the exhausting jets. Jet centerlines and decay characteristics were obtained from a total head rake. To obtain the three types of stratified exit flow characteristics considered in this study, the nozzles of figure 2 were utilized. The air supply for the core and annular regions of the dual concentric nozzle could be controlled independently to yield an annular jet with a high or a low velocity core.

### Equivalent Ideal Nozzle

The procedure which has been developed for determining an effective  $\dot{m}$  and  $d_o$  for nozzles with stratified exit flow characteristics is to consider a nozzle of the same mass flow and thrust, but having a uniform exit velocity profile.

The mass flow  $\dot{m}$  for an ideal nozzle of area  $A$  is

$$\dot{m} = A \sqrt{\frac{\gamma}{RT_o}} p M \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{1/2} \quad (5)$$

where  $p$  and  $M$  are the static pressure and Mach number of the flow at the nozzle exit, respectively.

The thrust  $F$ , assuming subsonic flow, is

$$F = 2Aq \quad (6)$$

where  $q$  is the nozzle exit dynamic pressure. Equation (5) may be written in terms of the dynamic pressure, so that

$$\dot{m}^2 = \frac{2A^2 q p}{RT_o} \left( 1 + \frac{(\gamma-1)q}{\gamma p} \right) \quad (7)$$

Eliminating  $A$  between equations (6) and (7) we deduce

$$q = \frac{p \left( \frac{F}{\dot{m}} \right)^2}{2RT_o - \frac{\gamma-1}{\gamma} \left( \frac{F}{\dot{m}} \right)^2} \quad (8)$$

and then, substituting for  $q$  in equation (6), we obtain

$$A = \frac{F \left( 2RT_o - \frac{\gamma-1}{\gamma} \left( \frac{F}{\dot{m}} \right)^2 \right)}{2p \left( \frac{F}{\dot{m}} \right)^2} \quad (9)$$

Thus,  $q$  and  $A$  for the ideal nozzle may be determined by substituting the nozzle mass flow and thrust in equations (8) and (9). The parameter  $m$  is then determined from the expression

$$m = \sqrt{\frac{q}{q_{\infty}}} \quad (10)$$

$q_{\infty}$  being the dynamic pressure of the crossflow. The diameter  $d_o$  used for normalizing purposes is determined from equation (9). It is

$$d_o = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{2F \left( 2RT_o - \frac{\gamma-1}{\gamma} \left( \frac{F}{\dot{m}} \right)^2 \right)}{\pi P \left( \frac{F}{\dot{m}} \right)^2}} \quad (11)$$

#### Annular Nozzle with High Velocity Core

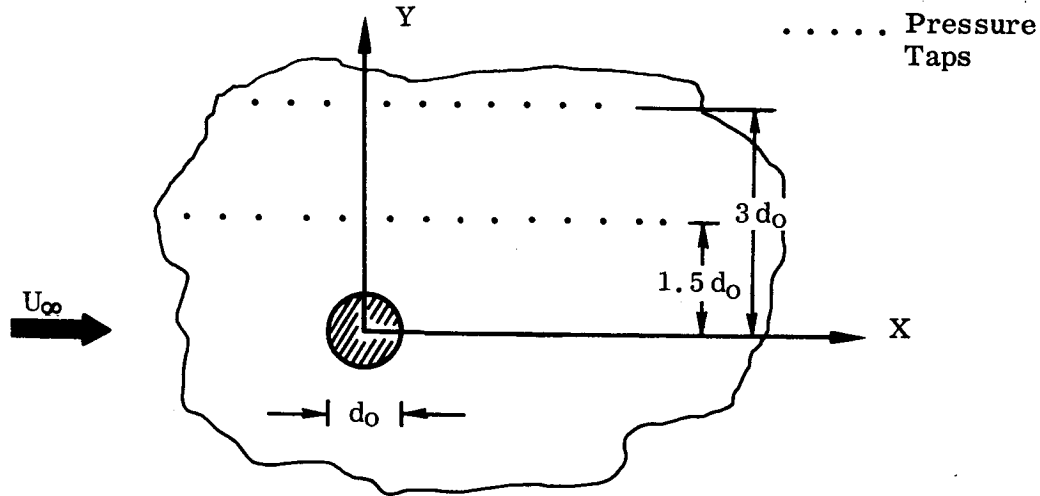
By considering an ideal nozzle, of the same mass flow and thrust, the effective jet dynamic pressure and effective diameter for the high velocity core nozzle tested in the related wind tunnel investigation (reference 16) have been determined. The core of the nozzle had an area of 0.8026 sq cm (0.1244 sq in), the annular region had an area of 4.0948 sq cm (0.6347 sq in). The dynamic pressures of the core and annular regions were 7.0931 N/cm<sup>2</sup> (1481.36 psf) and 2.7409 N/cm<sup>2</sup> (572.42 psf), respectively. Thus, assuming the jet total temperature to be ambient and the jet to exhaust into a standard atmosphere at sea level, we obtain

$$q_{je} = 3.5824 \text{ N/cm}^2 \text{ (748.18 psf)}$$

$$d_e = 2.4518 \text{ cm (0.9653 in)}$$

The effective diameter is not too different from the outer diameter of the nozzle, which is 2.54 cm (1.0 in), whereas the effective jet exit dynamic pressure is significantly different from both the core and annular values. The entrainment characteristics for the high velocity core nozzle, shown in figure 3, are not significantly different from those for the other stratified nozzles or for a convergent nozzle.

A sketch of the single-jet configuration tested in this investigation, showing the spanwise pressure tap stations at which comparisons between test data and calculations have been made, is shown below.



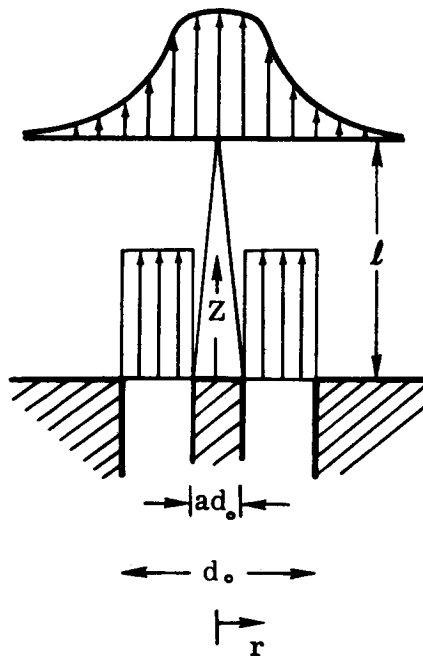
SKETCH 1

The theoretical prediction for the jet centerline is shown in figure 4. This may be compared with the test data for the high velocity core nozzle centerline shown in figure 5. Calculations of the surface pressure distribution compared with test data for the high velocity core nozzle, as well as for a clean nozzle at the same velocity ratio, are shown in figure 6.

It should be pointed out that, while comparisons between analytical results and experimental data are shown for only a representative number of jet configurations (and generally restricted to two spanwise stations for each configuration), this should not be viewed as a limit on the range of data acquired in the experimental phase of this investigation. Complete and extensive documentation of the wind tunnel test program may be found in reference 16.

### Annular Nozzle with Low Velocity Core

The singular case of an annular nozzle with a low velocity core, namely a dead air core, has been investigated. The dual concentric nozzle of figure 2 was utilized with no air being supplied to the core region. A schematic representation of the velocity profiles at the exit and at the end of the core region is shown in sketch 2. Details on the actual velocity profiles and the decay of the jet dynamic pressure may be found in reference 16.



SKETCH 2

After establishing the effective jet exit dynamic pressure for the nozzle, the tunnel (or freestream) dynamic pressure was adjusted to give the appropriate jet exit velocity ratio, and induced surface static pressure data were acquired. Figures 48 through 50 of reference 16 show that the experimental data for induced surface static pressures around the annular jet with dead air core collapse on the data for a uniform jet of the same velocity ratio very well when the effective jet exit diameter of the annular nozzle is used in nondimensionalizing distances associated with those data, except in the wake region behind the jet.

Thus, the concept of utilizing an effective  $m$  and  $d_o$ , obtained from considering an equivalent ideal nozzle as discussed previously, in the analytical model of reference 2 is seen to be a valid approach to treating velocity stratification effects for the annular nozzle with low velocity core in computing induced surface static pressures around the jet in those regions where the model of reference 2 is expected to yield meaningful results. The singularity representation of the jet used to compute the jet-induced pressure distribution in reference 2 assumes potential flow in the areas external to the jet and constitutes a good representation only in those regions where the flow outside the boundary layer is potential.

As an improvement to the equivalent ideal nozzle approach, the analysis of reference 2 may be modified for the annular jet with dead air core so that the internal mixing which takes place in the jet core may be properly accounted for.

Consider the annular jet of sketch 2. Let the diameter of the core, which is a region of no efflux ( $q_{j0}=0$ ), be  $a d_o$ , and the outer diameter of the annulus be  $d_o$ , and let the length of the core region be  $\ell$ . The diameter of the core at distance  $Z$  from the nozzle is  $a d_o - bZ$ , where  $b = a d_o / \ell$ . The equations of motion for a single jet exhausting into a crossflow are given by equations (2) through (4).

In the original analysis of uniform turbulent jets originating from circular nozzles, a development region was assumed in which the jet deformed from a circular cross section into an elliptical one. It is assumed here that this development region is not changed by the core region and that, in turn, the core region is independent of crossflow velocity.

Thus in the core region,  $0 \leq Z^* \leq a/b$ , we obtain

$$\frac{A_j}{d_o^2} = \frac{\pi}{4} d^{*2} \left[ 1 - \frac{5 Z^*}{2m} - \frac{1}{d^{*2}} (a^2 - 2ab Z^* + b^2 Z^{*2}) \right] \quad (12)$$

for the development region,  $Z^* \leq 0.3m$ , and

$$\frac{A_j}{d_o^2} = \frac{\pi d^{*2}}{16} \left[ 1 - \frac{4}{d^{*2}} (a^2 - 2ab Z^* + b^2 Z^{*2}) \right] \quad (13)$$

for the developed region,  $Z^* > 0.3m$ . Outside of the core region,  $Z^* > a/b$ , the expressions for  $A_j/d_o^2$  are as above with  $a = b = 0$ .



The expressions for the jet circumference,  $C$ , are not changed by the inclusion of the core term, so that

$$\frac{C}{d_o} = \pi d^* \left[ \frac{1 + \left[ 1 - \left( \frac{5}{2} \right) \left( \frac{Z^*}{m} \right) \right]^2}{2} \right]^{1/2} \quad \{Z^* \leq 0.3m\} \quad (14)$$

$$\frac{C}{d_o} = 2.24 d^* \quad \{Z^* > 0.3m\} \quad (15)$$

Equations (2) through (4) then become

$$\frac{dU_j^*}{dZ^*} = \frac{E}{\rho U_\infty d_o} \cdot \frac{\left[ \sin \theta - m U_j^* \right]}{\cos \theta \left( \frac{A_j}{d_o^2} \right) m^2 U_j^*} \quad (16)$$

$$\frac{d^2 X^*}{dZ^{*2}} = \frac{\left[ 1 + (X^*)^2 \right]^{3/2}}{\left( \frac{A_j}{d_o^2} \right) m^2 U_j^{*2}} \cdot \left[ \frac{E}{\rho U_\infty d_o} \cos \theta + \frac{C_D}{2} \cos^2 \theta d^* \right] \quad (17)$$

$$\begin{aligned} \frac{dd^*}{dZ^*} = & \left[ \left( \frac{E}{\rho U_\infty d_o} - m \cos \theta \left( \frac{A_j}{d_o^2} \right) \frac{dU_j^*}{dZ^*} \right) / m \cos \theta \right. \\ & \left. + 5\pi U_j^* \frac{d^{*2}}{8m} - \frac{\pi}{2} U_j^* (ab - b^2 Z^*) \right] / \frac{\pi}{2} U_j^* d^* \left( 1 - \frac{5Z^*}{m} \right) \end{aligned} \quad (18)$$

$$\frac{dd^*}{dZ^*} = \frac{\left[ \frac{E}{\rho U_\infty d_o} - m \cos \theta \left( \frac{A_j}{d_o^2} \right) \frac{dU_j^*}{dZ^*} + \frac{\pi}{2} U_j^* m \cos \theta (-ab + b^2 Z^*) \right]}{\frac{\pi}{8} U_j^* m \cos \theta d^*} \quad (19)$$

where the parameter  $m$  for this nozzle is now defined to be

$$m = \sqrt{\frac{\text{mainstream dynamic pressure}}{\text{annular jet dynamic pressure}}}$$

Equations (16) and (17) are valid for both the development region and the developed region of the jet. Equation (18) is to be used for the development region of the jet and equation (19) is valid for the fully developed region. Outside the core region,  $Z^* > a/b$ , equations (18) and (19) are modified by setting  $a=b=0$ .

Equations (1), (12) - (19), together with the jet exit boundary conditions, are the equations governing the jet development. They may be integrated, following reference 12, to obtain  $U_j^*$ ,  $d^*$  and  $X^*$  as functions of  $Z^*$ .

The annular jet which was tested in this investigation had an outer diameter of 2.54 cm (1.0 in) and a core diameter of 1.02 cm (0.40 in). The jet exit dynamic pressure was 7.09 N/cm<sup>2</sup> (1481 psf) for the annular region and the core region extended over a length of 10.16 cm (4.0 in). The parameters  $a$ ,  $b$  are then 0.40 and 0.10, respectively.

The computed jet centerline for the annular nozzle at a velocity ratio of 0.125 is shown in figure 4. The corresponding test data are shown in figure 5. From these results it is observed that the annular jet does not penetrate into the crossflow as far as a high velocity core nozzle (or clean nozzle) at the same velocity ratio.

Surface static pressure calculations are shown in figure 7, with the test data also shown for comparison. Good correlation between theory and test data is observed. Comparison with figure 6 indicates that the annular nozzle induces surface pressures of slightly smaller magnitude than the high velocity core nozzle (or clean nozzle).

### Vaned Nozzle

The vaned nozzle which has been considered in the experimental phase of this investigation is shown in figure 2. The induced surface static pressure data plotted in figures 48 and 49 of reference 16 (again utilizing for nondimensionalizing purposes the effective jet exit diameter for the vaned nozzle, obtained from equivalent ideal nozzle considerations) indicate that the induced pressure distributions around a vaned jet may be predicted quite accurately by using the equivalent ideal nozzle approach.

The presence of the vanes in the nozzle reduces the exit area and one might expect a change in the mixing characteristics for this nozzle. However, if the mass flow  $\dot{m}$  is plotted against distance, normalized by the effective diameter as defined in equation (11), the entrainment characteristics of all the nozzles are similar (figure 3). It may be deduced, therefore, that the vanes only affect the internal mixing of the jet,

so that the modified approach developed for the annular nozzle with dead air core, to account for the internal mixing taking place in the jet core, may serve as an improvement over the equivalent ideal nozzle approach for the vaned jet as well. The area of the vanes at the nozzle exit is replaced by a circular core of equal area and the extent of the core region is again determined from static test results (reference 16).

The nozzle which was tested in this study had a vaned area of 1.719 sq cm (0.266 sq in), yielding a core diameter of 1.48 cm (0.58 in). The extent of the core region was determined to be 14.73 cm (5.8 in). The parameters  $a$  and  $b$  of equation (12) are, therefore, 0.58 and 0.10, respectively. The parameter  $m$  for this nozzle was defined as

$$m = \sqrt{\frac{\text{mainstream dynamic pressure}}{\text{jet exit maximum dynamic pressure}}}$$

Computed jet centerlines for the vaned nozzle, at velocity ratios of 0.125 and 0.250, are shown in figure 4. The corresponding test data are shown in figure 5. Figure 5 includes test data for the vanes perpendicular to the crossflow and for the vanes aligned with the crossflow. The calculations of figure 4 do not, of course, account for this difference in the alignment of the vanes. The vaned nozzle jet is observed to penetrate the mainstream less than either the annular nozzle jet or the high velocity core jet at the same velocity ratio.

Figure 8 shows computed surface pressure distributions for the vaned nozzle, with the vanes perpendicular to the crossflow, at a velocity ratio of 0.125. Test data are included for comparison. The correlation is observed to be quite good. The pressure data for this nozzle, both calculated and test, are observed to be of slightly lower magnitude than those for the annular nozzle (figure 7), at the same velocity ratio. Computed and experimental surface pressure distributions for the vaned nozzle, with the vanes aligned with the crossflow, are shown in figure 9. The velocity ratio is again 0.125 and the computed pressure distributions are, of course, the same as those shown in figure 8. No significant effect due to the changing of the orientation of the vanes is observed from the test data.

Figure 10 shows pressure distributions around the vaned nozzle jet, with the vanes perpendicular to the crossflow, at a velocity ratio of 0.250. The test data from two runs using different values of jet exit dynamic pressure and freestream dynamic pressure to achieve the velocity ratio are included. No significant differences between these two sets of data are observed.

## MUTUAL INTERFERENCE EFFECTS

The single-jet model of reference 2 was extended in reference 12 to treat jets exhausting into arbitrarily directed freestreams, as well as multiple-jet configurations.

Multiple-jet configurations were treated as combinations of discrete jets, with leading jets assumed to develop independently and downstream jets assumed to exhaust into a crossflow of reduced dynamic pressure. The test data of reference 13 were used to derive an expression for the effective dynamic pressure which the downstream jet "sees" as a result of the blockage of the crossflow by the upstream jet. This expression was based on data for two-jet configurations at zero sideslip (jet exits aligned in the freestream direction), with both jets exhausting normally into the freestream.

The test data were utilized to verify that the upstream jet develops independently of the downstream jet for the zero sideslip condition, even for the closely spaced two-jet configuration tested in reference 13. For the closely spaced configuration at zero sideslip, the downstream jet "sees" a low crossflow dynamic pressure and, consequently, does not exert a strong influence on the induced flow field. The assumption that the upstream jet develops independently of the downstream jet is therefore justified, despite the close jet spacing. Since the expression for the effective crossflow dynamic pressure for the downstream jet was based on data for the zero sideslip configurations, it accounts for all the interference effects between the two jets. The good agreement between calculated induced pressure distributions and the test data for the zero sideslip configurations, exhibited in the comparisons of reference 12, supports this conclusion.

For the closely spaced configuration at sideslip, noticeable differences between theory and test data were evident. With the jets no longer aligned in the freestream direction, the downstream jet now has a stronger influence on the induced flow field since there is less blockage of the crossflow by the upstream jet. This stronger influence, together with the close jet spacing, makes the assumption that the upstream jet develops independently of the downstream jet no longer representative of the physical situation. It was felt that further mutual interference effects between the two jets had to be included to improve correlation between theory and test data.

The extension of the basic two-jet computation to include mutual interference effects between the two jets, in addition to the blockage effect discussed previously, is presented here. Comparisons between theory and test data of reference 13 for the closely spaced two-jet configuration are shown.

The modification of the expression for the crossflow dynamic pressure which the downstream jet "sees" in a two-jet configuration, with both jets exhausting at an angle other than  $90^\circ$  into the crossflow, is discussed. Calculations of jet centerlines and induced surface static pressures are compared with test data of reference 16 for three different spacings between two inclined jets.

### Two-Jet Analytical Model

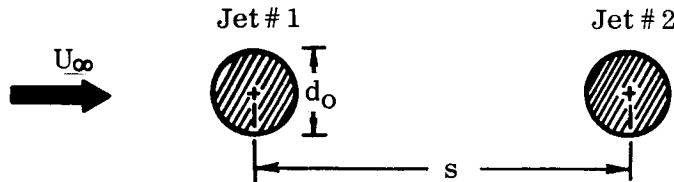
The details of the computational procedure applying the basic single-jet model to the calculation of the interaction flow field due to two exhausting jets are given in reference 12.

A two-jet configuration was treated as a combination of discrete jets. The equations of motion for each of the exhausting jets were integrated, utilizing the appropriate initial conditions for each jet,  $Z^* = 0.$ ,  $X^* = 0.$ ,  $U_j^* = 1.$ ,  $d^* = 1.$ , and, using the coordinate system of figure 1,  $dX^*/dZ^* = [(1. - \cos^2 \theta_o) / \cos^2 \theta_o]^{1/2}$ , as well as the corresponding jet exit velocity ratio, to yield the mean jet speed  $U_j^*$ , the major diameter of the ellipse representing the jet cross section  $d^*$ , and the displacement of the jet centerline in the freestream direction  $X^*$ , all in nondimensionalized form, as functions of  $Z^*$ , the nondimensionalized penetration of the jet centerline into the crossflow.

The upstream jet was assumed to develop independently of the downstream jet and the downstream jet was assumed to exhaust into a crossflow of reduced dynamic pressure, which it "sees" as the result of blockage by the upstream jet. Thus the influence of the upstream jet on the downstream jet was introduced into the computations as a reduced freestream velocity,  $U_e/U_\infty = [q_e/q_\infty]^{1/2}$ , in the equations governing the development of the downstream jet (equations (2) - (4)).

The extent of overlap between the two jets was the principal parameter in determining the degree of blockage experienced by the downstream jet. The computational details of establishing this degree of influence of the upstream jet on the downstream jet for each jet element, as the integration of the equations of motion is being carried out, are given in reference 12.

Test data of reference 13 were used to obtain an empirical relationship for the dynamic pressure  $q_e$  which the downstream jet "sees" as the result of the crossflow blockage by the upstream jet, in terms of the crossflow dynamic pressure,  $q_\infty$ , and the spacing between the two jets,  $s$  (see sketch below).

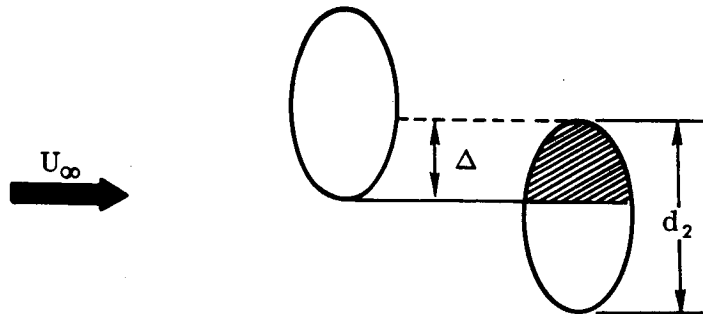


SKETCH 3

This expression is given in reference 12 as

$$\sqrt{\frac{q_e}{q_\infty}} = \frac{s/d_0 - 1}{s/d_0 + .75} \quad \{s/d_0 > 1\} \quad (20)$$

and is used as a limiting value, when computation of the overlap between the two jets shows that the downstream jet element is completely in the zone of influence of the upstream jet element (as, for example, in the case of two jets aligned in the crossflow direction). When the two jets are not aligned, an effective crossflow dynamic pressure,  $q_\beta$ , which is a weighted mean of  $q_e$  given above and  $q_\infty$ , is utilized. The weighting of the dynamic pressure is determined from the degree of overlap between the upstream and the downstream jet elements discussed previously and shown in schematic form below.



SKETCH 4

Thus,

$$\sqrt{q_\beta} = \frac{\Delta \sqrt{q_e} + (d_2 - \Delta) \sqrt{q_\infty}}{d_2} \quad (21)$$

As the equations of motion for the two exhausting jets are being integrated, the distance between the two jet centerlines is continually checked. When intersection of the two jets is indicated, initial conditions for the merged jet which results are determined from continuity and momentum considerations, as detailed in reference 12. These initial conditions are then employed in integrating the set of differential equations for  $U_j^*$ ,  $d^*$ , and  $X^*$ .

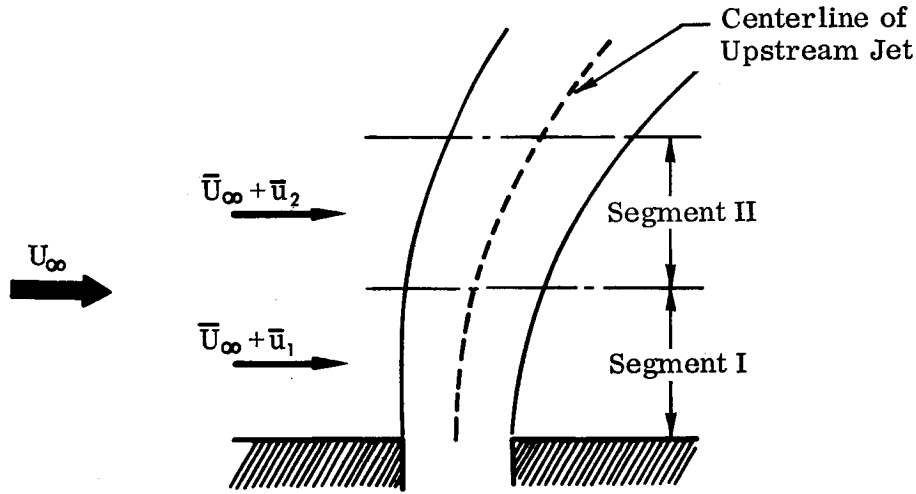
The velocity field induced by a two-jet configuration can now be determined by replacing each jet (including the jet resulting from the coalescence of the two exhausting jets) by its representative singularity distribution of sinks and doublets.

The expression for the dynamic pressure to be utilized in the downstream jet computations (equation (20)) was based on data for two-jet configurations at zero sideslip, with both jets exhausting normally into the crossflow, and contains only the jet exit spacing as a parameter.

Since equation (20) was based on data for zero sideslip configurations, it accounted for all the interference effects between the two jets, even for the closely spaced configuration,  $s = 2.5d_0$ , of reference 13. Comparisons of test data with theoretical predictions in reference 12 showed good correlation for the zero sideslip configurations (see, for example, figures 37 and 40 of reference 12).

Noticeable differences between theory and test data were discernible for the non-zero sideslip configurations, particularly for the close jet spacing (see, for example, figure 41 in reference 12). For these computations  $q_e$  as given by equation (20) is weighted with  $q_\infty$  according to equation (21). The downstream jet now has a stronger influence on the induced flow field, and it was felt that, although the relationship of equation (21) accounts for the effect of blockage of the upstream jet on the downstream jet, further mutual interference effects between the two jets had to be included in the computations to improve the correlation between theory and test data.

For the purpose of including mutual jet interference effects, an iterative procedure involving modification of the crossflow into which the two jets exhaust, has been chosen. The scheme treats each of the exhausting jets in a number of segments as shown in sketch 5.



SKETCH 5

The freestream velocity vector for each segment of one exhausting jet is perturbed by the induced velocity vector ( $\bar{u}_1$ ,  $\bar{u}_2$ , etc) due to the other exhausting jet. The perturbed freestream velocity vector is assumed constant over the extent of the segment and is evaluated at the point of origin of each segment. Each segment is treated as a discrete jet, with proper initial conditions and the appropriate freestream velocity vector.

The first computation sets  $\bar{u}_1$ ,  $\bar{u}_2$ , etc, equal to zero to establish the first approximation for the centerlines of the two exhausting jets and the coalesced jet, if intersection between the two jets occurs. The two exhausting jets and the coalesced jet are then replaced by their representative singularity distributions and the induced velocities  $\bar{u}_1$ ,  $\bar{u}_2$ , etc, are then computed for each segment of the two exhausting jets.

For the first iteration, the initial conditions for segment I now become

$$d_1^* = 1., \quad U_{j1}^* = U_{j1}/U_{j01} = 1., \quad m_1 = U_{j01}/U_\infty$$

where  $m_1$  is the inverse velocity ratio of the exhausting jet.

The direction cosines of the modified freestream velocity vector,  $\bar{U}_\infty + \bar{u}_1$ , are determined and a local coordinate system is established which is aligned with the freestream velocity vector and the jet exhaust vector (see reference 12, page 25). The initial value for  $dX^*/dZ^*$  is determined in this local coordinate system.



The effective inverse velocity ratio for segment I is

$$m_{1\text{eff}} = \frac{U_{j01}}{U_\infty} \left( \frac{U_\infty}{U_\infty + u_1} \right) = m_1 \left( \frac{U_\infty}{U_\infty + u_1} \right)$$

The equations for  $U_j^*$ ,  $d^*$ ,  $X^*$  may now be integrated over the extent of segment I.

The last point of segment I then becomes the origin of the next segment (or next discrete jet) with a diameter  $d_{02} = d_1^* d_{01}$ , where  $d_1^*$  is the last computed value of the nondimensionalized jet diameter in segment I

Other initial conditions for segment II are

$$d_2^* = 1., \quad U_{j2}^* = U_{j2}/U_{j02} = 1., \quad m_2 = U_{j02}/U_\infty = U_{j1}^* m_1$$

where  $U_{j1}^*$  is the last computed value of the nondimensionalized mean jet speed in segment I.

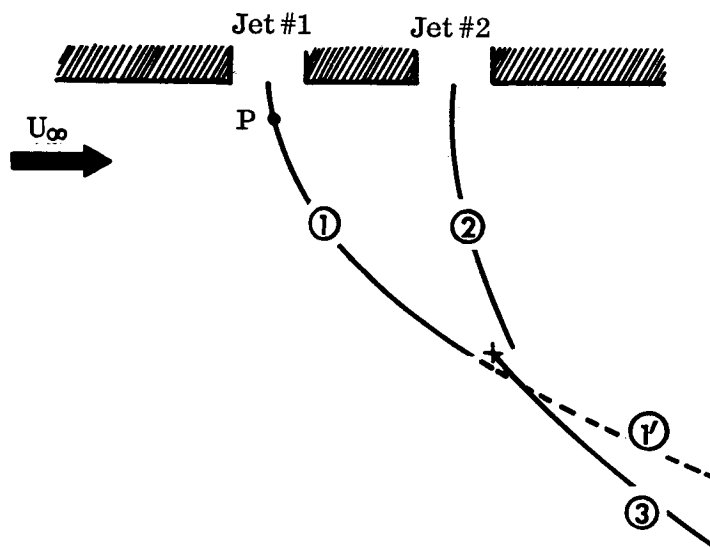
At this point the direction cosines of  $\bar{U}_\infty + \bar{u}_2$  are determined and a new local, jet-oriented coordinate system is established. The initial value for  $dX^*/dZ^*$  is determined in this coordinate system from the known direction of the jet centerline at the end of segment I. The effective inverse velocity ratio for segment II is

$$m_{2\text{eff}} = \frac{U_{j02}}{U_\infty} \left( \frac{U_\infty}{U_\infty + u_2} \right) = U_{j1}^* m_1 \left( \frac{U_\infty}{U_\infty + u_2} \right)$$

The equations for  $U_j^*$ ,  $d^*$ ,  $X^*$  may now be integrated over the extent of segment II.

The computations described above continue for each of the two exhausting jets until integration over the extent of each jet, up to the point of intersection with the other jet, has been accomplished. The procedure of establishing initial conditions for the jet resulting from coalescence of the two exhausting jets remains unchanged from that detailed in reference 12 and outlined previously. New values for  $\bar{u}_1$ ,  $\bar{u}_2$ , etc, may now be obtained and the entire computational scheme may be repeated.

In determining the induced velocity vectors  $\bar{u}_i$  for the segments of each exhausting jet, only the effect of the other jet is to be considered. In the representation of sketch 6, the singularity distributions of two exhausting jets, ① and ②, and of the coalesced jet ③ are indicated. The singularity distribution ①' is a continuation of ① and would result if no intersection occurred between Jet #1 and Jet #2.



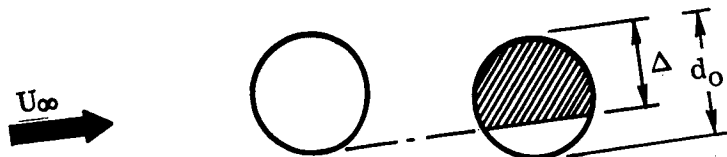
SKETCH 6

It is desired to evaluate the flow field induced at point P by Jet #2 alone. This is accomplished by summing the contributions to the induced velocity components at P from the segments constituting ② and adding the contributions due to the coalesced jet ③. But the coalesced jet is established from continuity and momentum considerations involving both Jet #1 and Jet #2 and thus, some influence of Jet #1 at point P is now included. The contributions to the induced flow field at P due to ①' are now subtracted to account for this, and the induced velocity components  $V_x$ ,  $V_y$ ,  $V_z$  are obtained.

As discussed previously, the application of equation (20) in the two-jet computations appeared to adequately account for all mutual interference effects for the zero sideslip configurations, since equation (20) was derived from zero-sideslip data. With increasing sideslip, equation (21) shows that  $q_\beta$  approaches  $q_\infty$  as  $\Delta$  (or the degree of overlap) approaches zero. For a spanwise configuration, the only interference effects between the two jets would be those accounted for by the modification of the crossflow by the iterative procedure.

Equation (21) then suggests that, after the induced velocity components  $V_x$ ,  $V_y$ ,  $V_z$  have been evaluated, the degree of overlap, as represented by the term  $(d_2 - \Delta)/d_2$ , be considered before the freestream velocity vector is modified.

As an approximation to the term  $(d_2 - \Delta)/d_2$  in equation (21), a factor  $K$  is evaluated, using properties in the plane of the jet exits, as shown in sketch 7, such that  $K = (d_0 - \Delta)/d_0$ .



SKETCH 7

The induced velocity vector  $\bar{u}_i$  is then obtained from the induced velocity components

$$\bar{u}_i = K (V_x \hat{i} + V_y \hat{j} + V_z \hat{k})$$

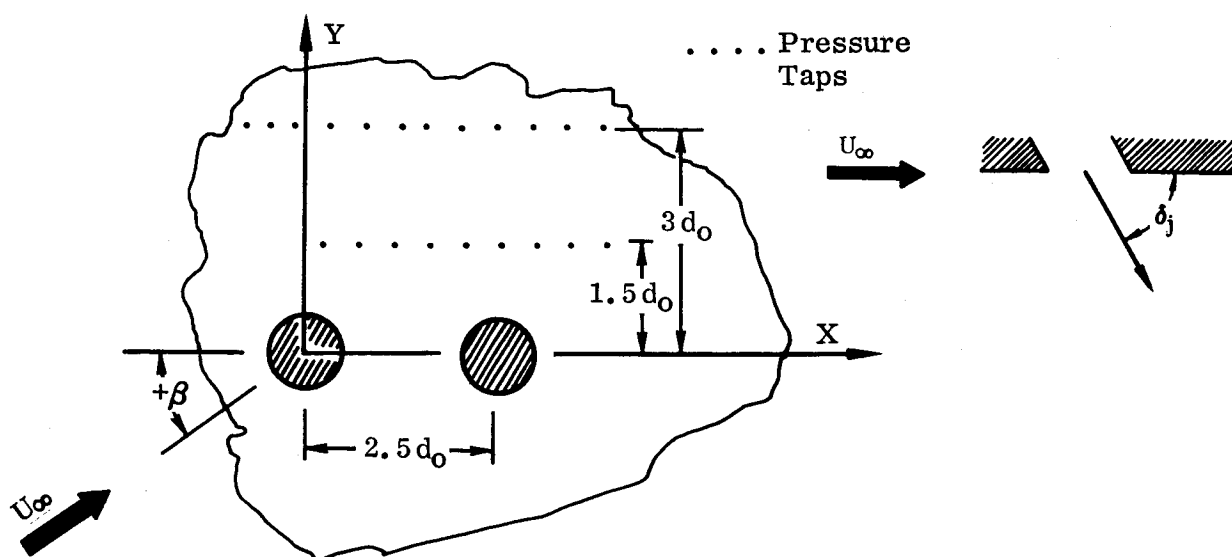
Thus, no further mutual interference effects are included for the zero sideslip configurations, and full effect of the modification of the crossflow is included for a configuration where equation (21) does not provide for any blockage effect.

#### Comparisons of Two-Jet Calculations With Test Data

Computations have been carried out primarily for the closely spaced two-jet configuration for which induced pressure distributions in the plane of the jet exits were obtained in reference 13. A schematic of this configuration is shown in sketch 8.

Figure 11 shows a comparison between theory and test data for induced pressure variation with  $X/d_0$  at  $Y/d_0 = 1.5$  and 3. Since mutual interference effects due to the modification of the crossflow are not included for a zero sideslip configuration, as discussed previously, the computed pressure distributions should agree with those presented in figure 40 of reference 12. Comparison with the theoretical pressure distributions of figure 40, reference 12, does confirm that numerical differences incurred by breaking the two exhausting jets into segments and treating the segments as discrete jets are negligible. This, of course, applies only to the computation of the initial approximation when  $\bar{u}_1$ ,  $\bar{u}_2$ , etc, are equal to zero.

Figure 12 shows the same comparison for a sideslip angle  $\beta = 20^\circ$ . Pressure

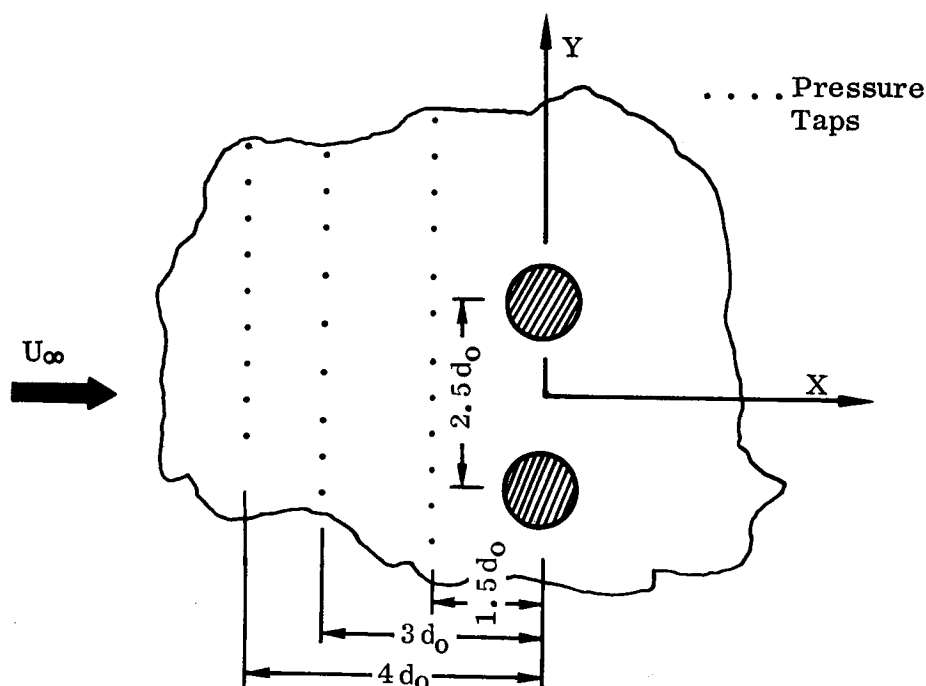


SKETCH 8

distributions corresponding to the first approximation and the final iteration of the computational scheme described previously are shown. Some improvement in correlation between theory and test data is discernible due to the incorporation of these further mutual interference effects.

For this configuration, the projections of the centerlines in the X-Y and the X-Z plane are shown in figure 13. In contrast to the results for  $\beta = 0$  (reference 12), the calculated centerline for the merged jet indicates greater penetration than observed in the wind tunnel test. Also, there is a significant difference between calculation and test data in the projection of the centerlines in the X-Y plane. The experimentally determined centerlines (positions of maximum total head) exhibit an unexpected lateral deflection, which may be due to the partial blockage of the downstream jet resulting in the jet momentum decaying at a decreased rate on the side of the jet which is blocked from the crossflow by the leading jet.

A two-jet configuration with a spanwise spacing of 2.5 diameters is shown in sketch 9. Theoretical and experimental pressure variations are shown for three stations of constant  $X/d_0$  in figure 14. Again, the first approximation and the final iteration for each computed pressure distribution are shown. The full effect of the mutually induced velocities on the crossflow is included in the iterative procedure for this configuration, resulting in noticeable improvement in the correlation between theory and test data.



SKETCH 9

The projections of the jet centerlines for this configuration, as well as for one with a spacing of 7.5 diameters between the jet exits, are shown in figure 15. For the wider spacing, the jets are attracted to each other and each is deflected by the crossflow to a greater extent than a single jet of the same velocity ratio. These features are predicted quite well by theory. For the close spacing, the analytical model predicts that each jet will be deflected to a greater extent (up to the point of intersection) than the jets with the wide spacing. Following intersection, this trend is reversed and the computed centerline of the merged jet exhibits greater penetration than the individual jet centerlines of the widely spaced configuration. As was the case for the closely spaced configuration at sideslip  $\beta = 20^\circ$ , the positions of maximum total head show an unexpected lateral deflection, in contrast to the calculation indicating intersection after a penetration of about 6 diameters.

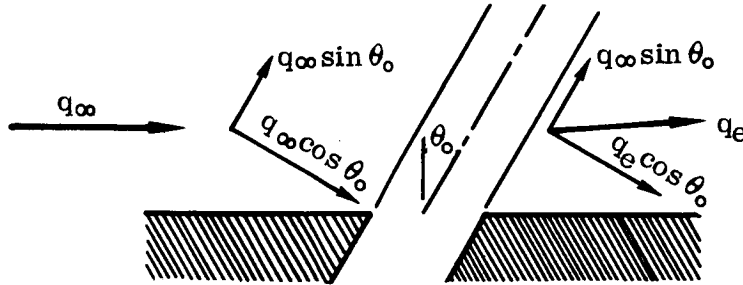
In computing the pressure distributions of figures 12 and 14, two iterations on mutual interference effects were employed after the initial approximation to each pressure distribution had been established. Indications are that two iterations are sufficient. Experience with the test cases has shown that the first iteration (establishing the induced velocities by which the crossflow is modified as other than zero) is primarily responsible for effecting the changes in the computed pressure distribu-

tions, and subsequent iterations produce little change. This is illustrated in figure 16 where the first and second iterations for the pressure distributions shown in figures 12 and 14 are presented.

### Multiple Inclined Jets

The blockage effect of the upstream jet on the downstream jet, as given by equation (20), is seen to be a function of the spacing between the two exhausting jets only. The expression was based on experimental data of reference 13, where all multiple-jet configurations tested consisted of jets exhausting normally into the crossflow.

It was felt that for an inclined jet exhausting into the crossflow, equation (20) represents a reduction in the crossflow velocity normal to the jet. There is also a component of the crossflow velocity tangential to the jet, as shown below.



SKETCH 10

Thus, the magnitude of the effective dynamic pressure  $q'_e$  is

$$\sqrt{\frac{q'_e}{q_\infty}} = \left[ \left( \frac{q_e}{q_\infty} \right) \cos^2 \theta_0 + \sin^2 \theta_0 \right]^{1/2} \quad (22)$$

where  $\sqrt{q_e/q_\infty}$  is given by equation (20).

Figures 17 through 23 show comparisons of theoretical predictions and test data from reference 16, for three different spacings of two inclined jets. Equation (22) was utilized to account for the effect of blockage on the downstream jet and the iterative procedure described previously was employed for including further mutual interference effects between the two jets in the closely spaced configuration with sideslip.

The jet centerlines are predicted quite well. After intersection, the merged jet shows greater penetration into the crossflow than is indicated by theory, as was the case for jets exhausting normally into the crossflow (reference 12). The induced pressure distributions (figures 20 through 23) exhibit features similar to those discernible in surface static pressure distributions due to two-jet configurations exhausting normally into the crossflow. For the close spacing, the pressure distribution resembles that induced by a single jet. As the spacing between the two jets increases, the downstream jet is seen to have its own discrete effect on the pressure distribution.

## CONCLUSIONS

The concept of utilizing an effective jet exit velocity and diameter, obtained by considering a nozzle of the same mass flow and thrust but having a uniform exit velocity profile (equivalent ideal nozzle), in conjunction with an analytical model for a uniform jet exhausting into a crossflow has been shown to be a valid approach for determining flow fields due to jets with exit velocity stratification. For the singular case of a low velocity core nozzle, namely a dual concentric nozzle with a dead air core, the analysis for a jet with uniform exit velocity profile has been modified to take into account the internal mixing in the jet core, as an improvement over the equivalent ideal nozzle approach. This modified analysis serves as an improvement over the equivalent ideal nozzle approach for the vaned jet as well.

The investigation has shown that induced surface static pressure distributions due to stratified jets exhausting into a crossflow are not appreciably affected by the details of the exit velocity stratification. This indicates that small scale testing may be accomplished with uniform exit velocity profile nozzles, without having to take recourse to reproducing, in detail, the stratified exit flow characteristics of lift/propulsive systems of V/STOL configurations.

Calculations of jet centerlines for the three types of nozzles investigated show that, for a given velocity ratio, the jet originating from a high velocity core nozzle penetrates the crossflow to the greatest extent, and the jet originating from the vaned nozzle exhibits the least penetration. These trends may be observed in the test data generated as part of this study.

Inclusion of mutual interference effects between jets in a two-jet configuration has improved the correlation between theory and test data for two closely spaced jets exhausting normally into the crossflow. For two jets exhausting at an angle of 60° into the crossflow, calculations of centerlines and induced surface static pressures are in good agreement with test data.



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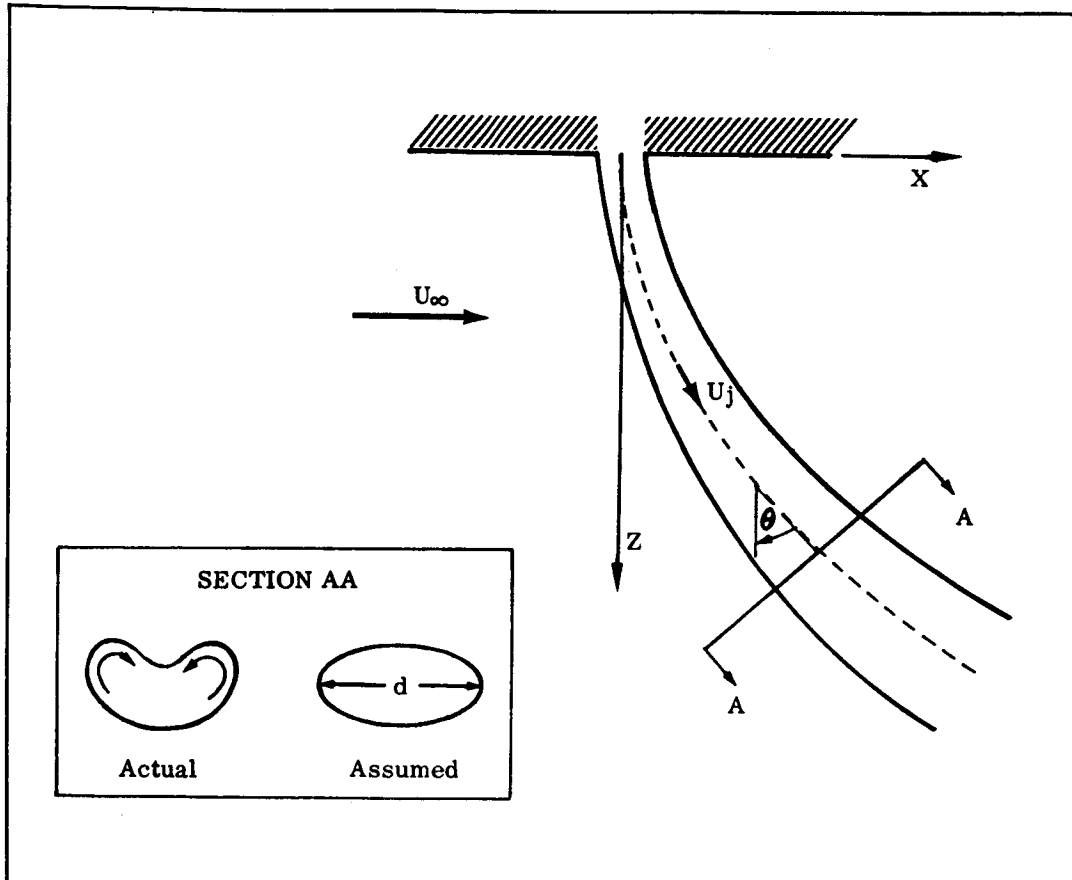
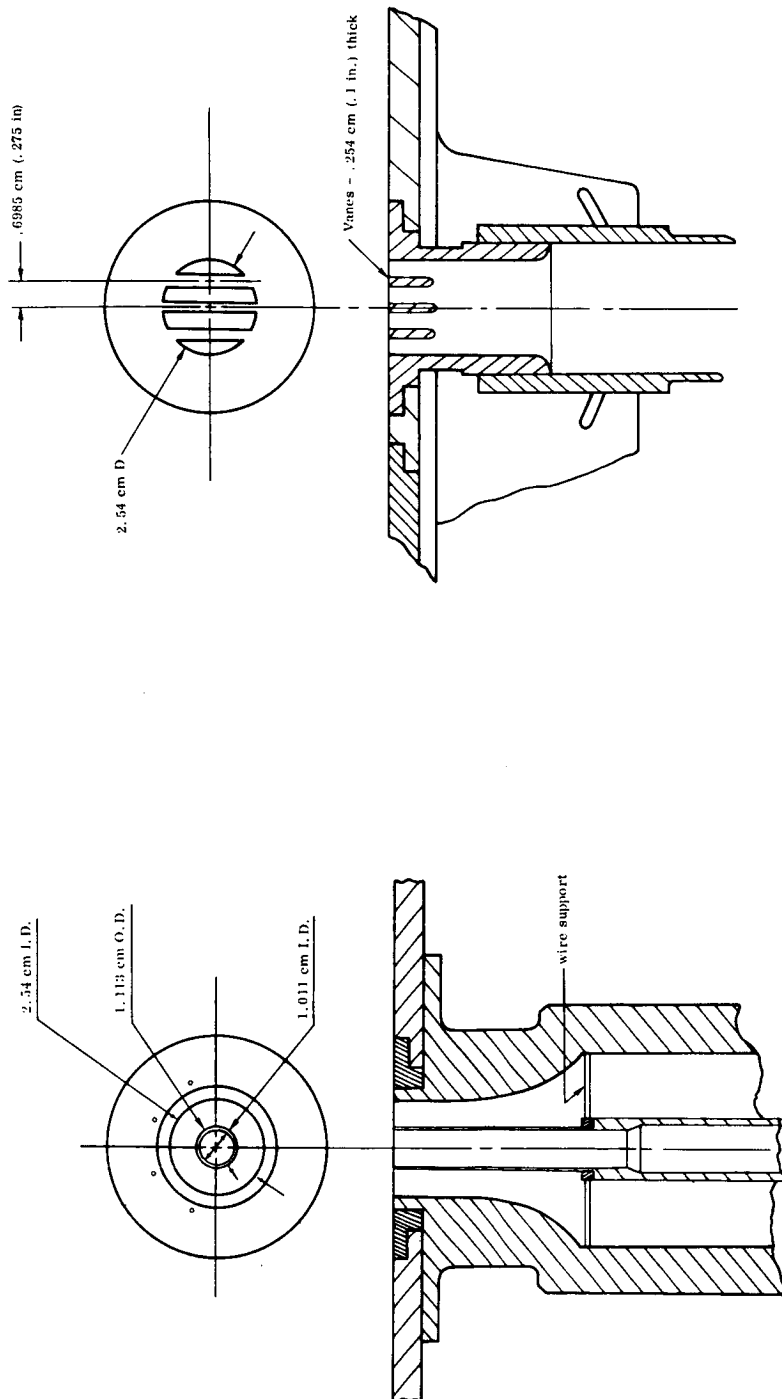


FIGURE 1. JET EXHAUSTING NORMALLY INTO THE FREESTREAM



(b) Vaned Nozzle

(a) Dual Concentric Nozzle

FIGURE 2. STRATIFIED FLOW NOZZLES

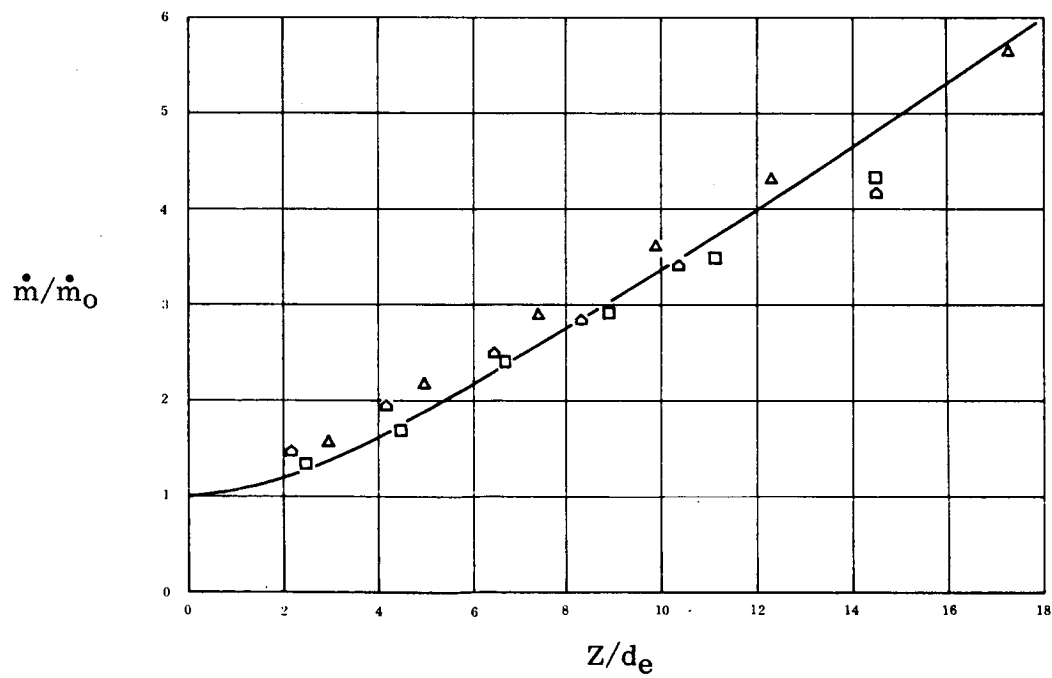
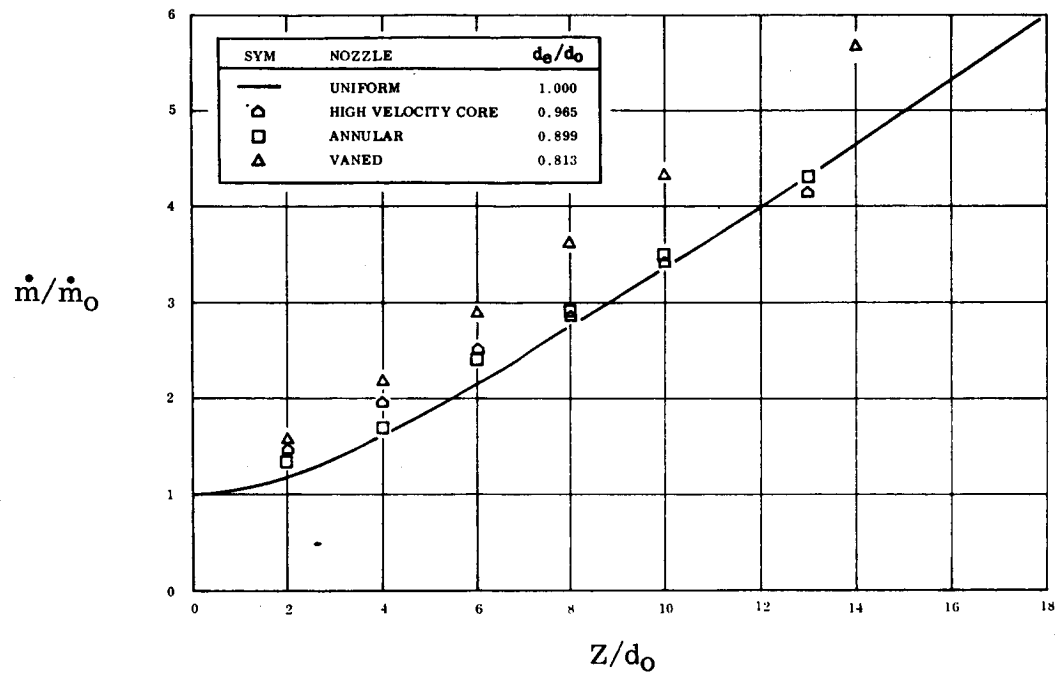


FIGURE 3. MASS FLOW FOR STRATIFIED NOZZLES

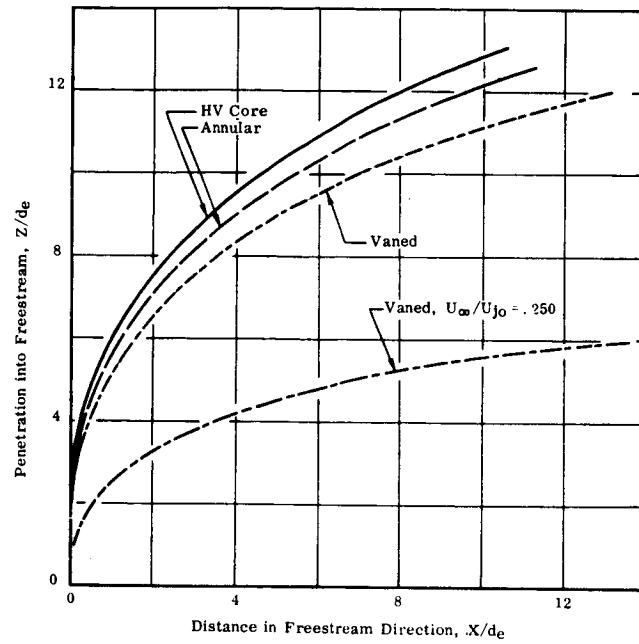


FIGURE 4. CALCULATED JET CENTERLINES ( $U_\infty/U_{j0} = 0.125$ )

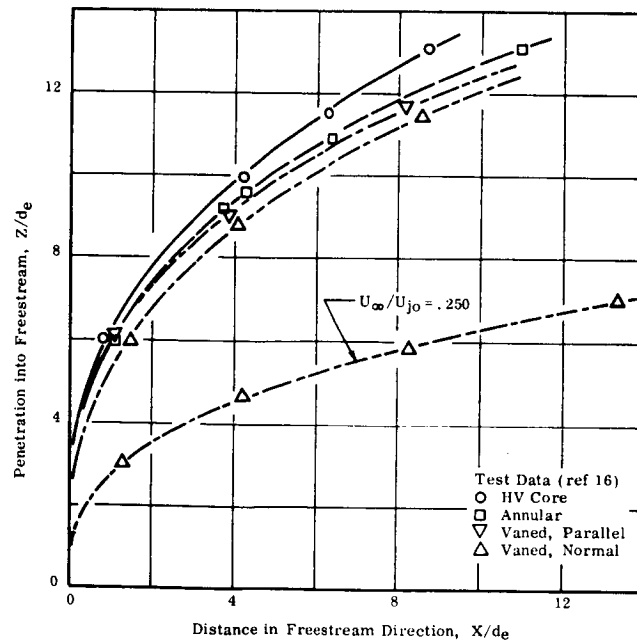
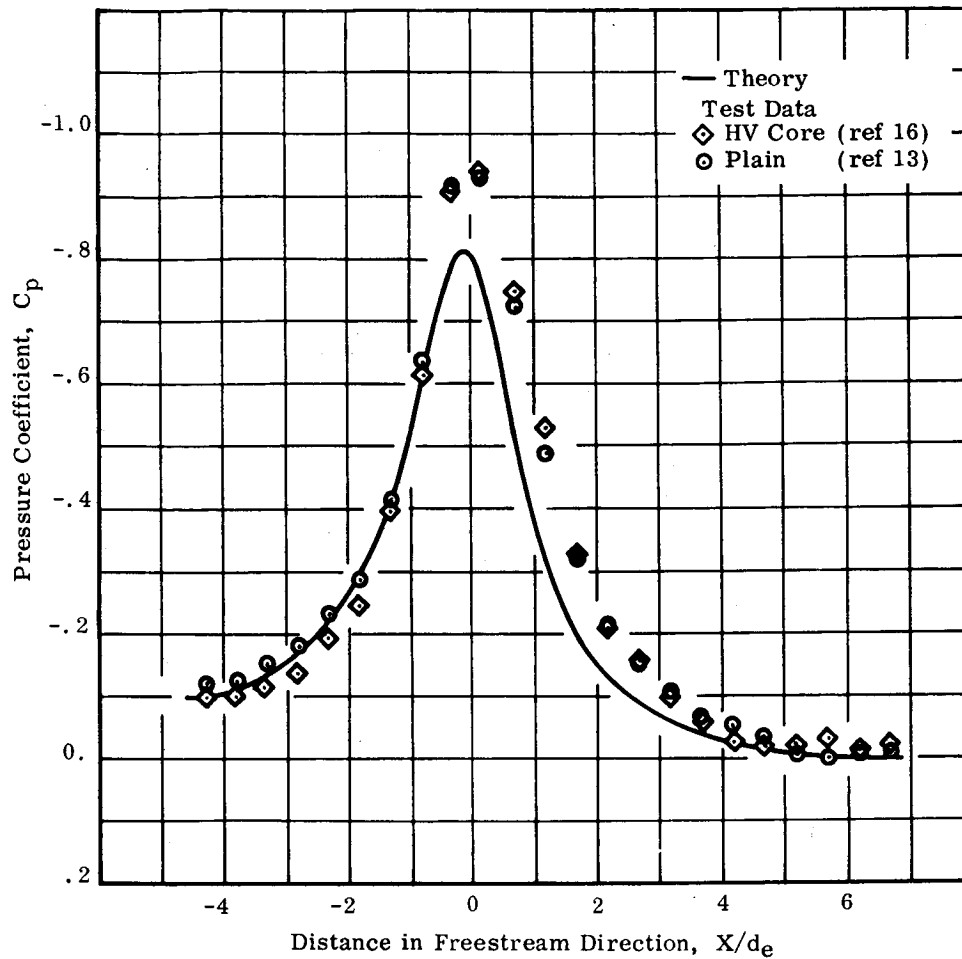
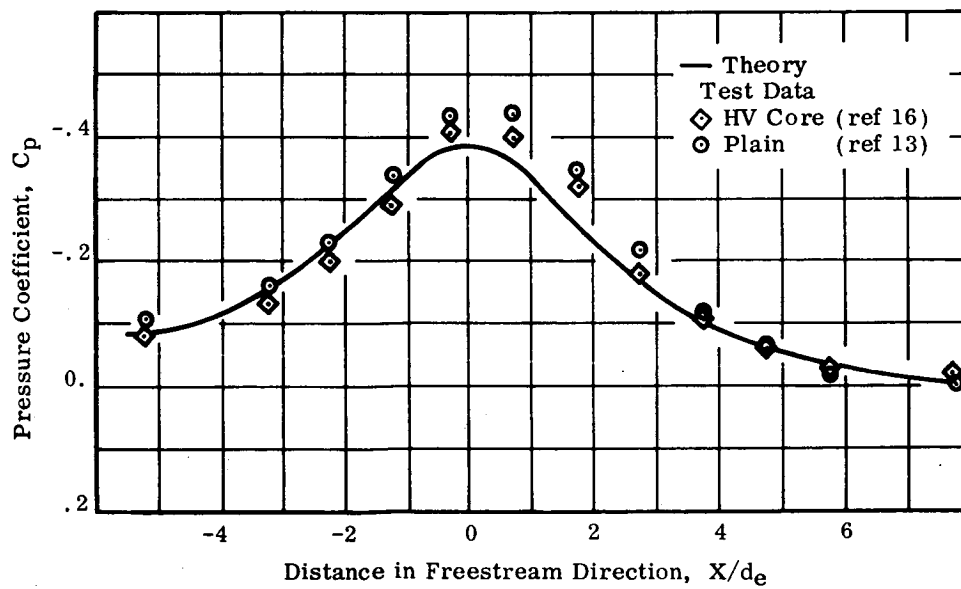


FIGURE 5. EXPERIMENTAL JET CENTERLINES ( $U_\infty/U_{j0} = 0.125$ )

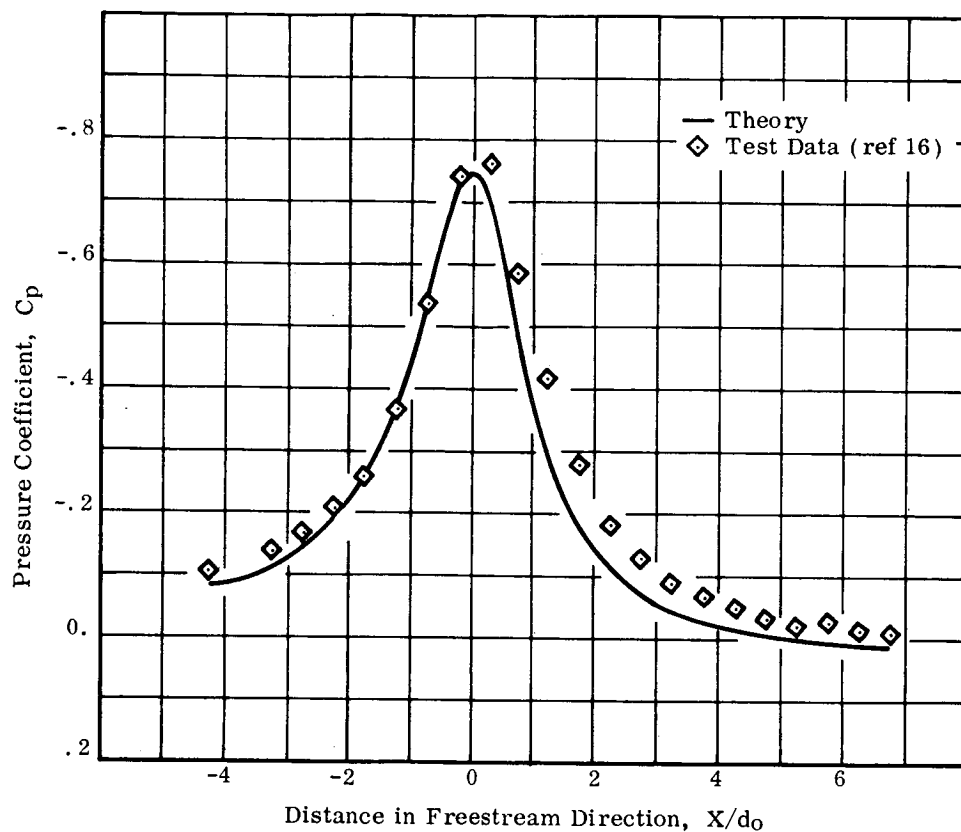


(a) Station  $Y/d_0 = 1.5$

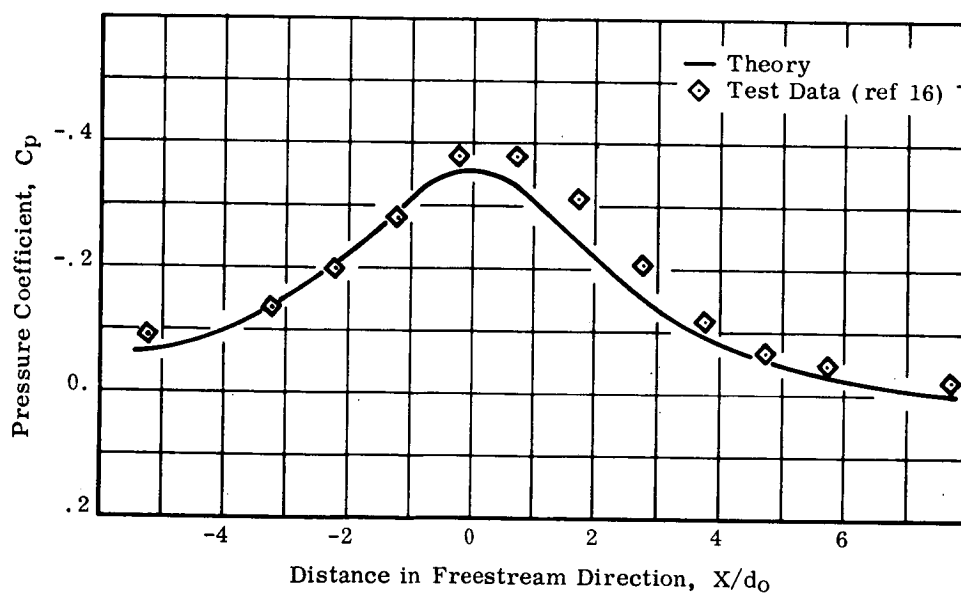


(b) Station  $Y/d_0 = 3$

FIGURE 6. INDUCED PRESSURE VARIATION FOR A HIGH VELOCITY CORE JET ( $U_\infty/U_{j0} = 0.125$ )

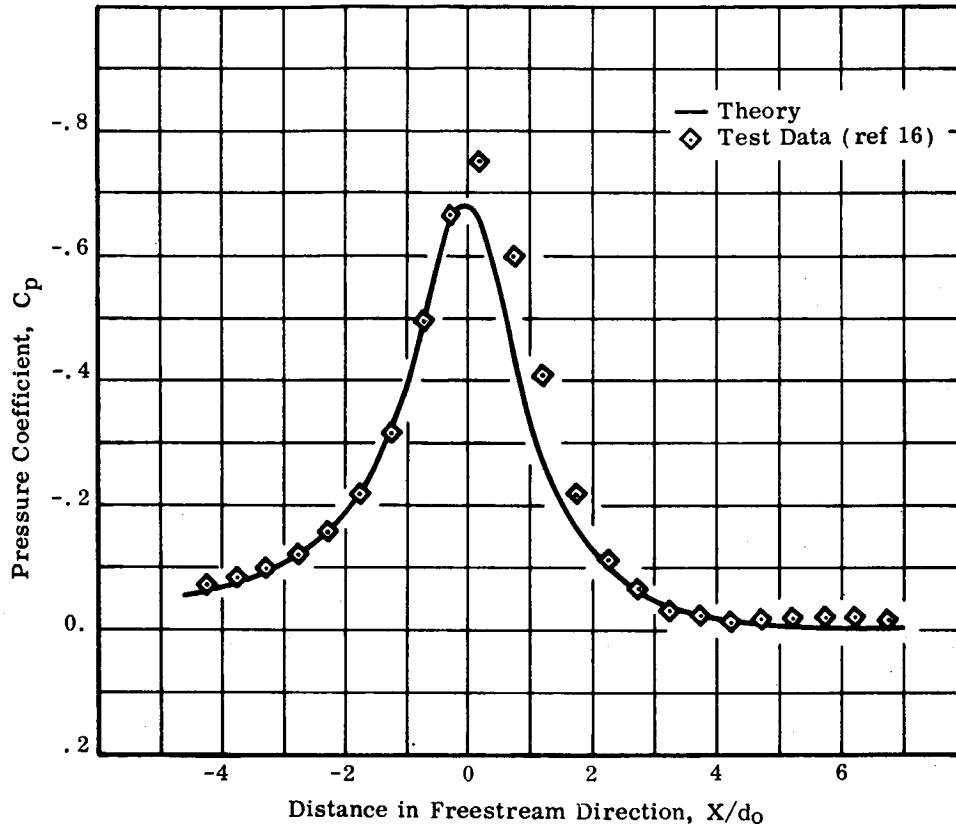


(a) Station  $Y/d_0 = 1.5$

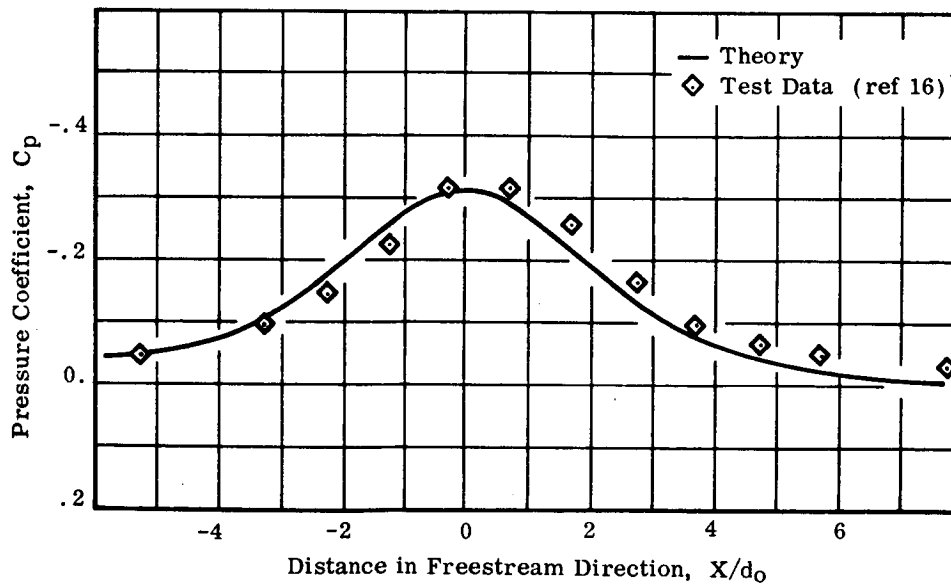


(b) Station  $Y/d_0 = 3$

FIGURE 7. INDUCED PRESSURE VARIATION FOR AN ANNULAR JET ( $U_\infty/U_{j0} = 0.125$ )



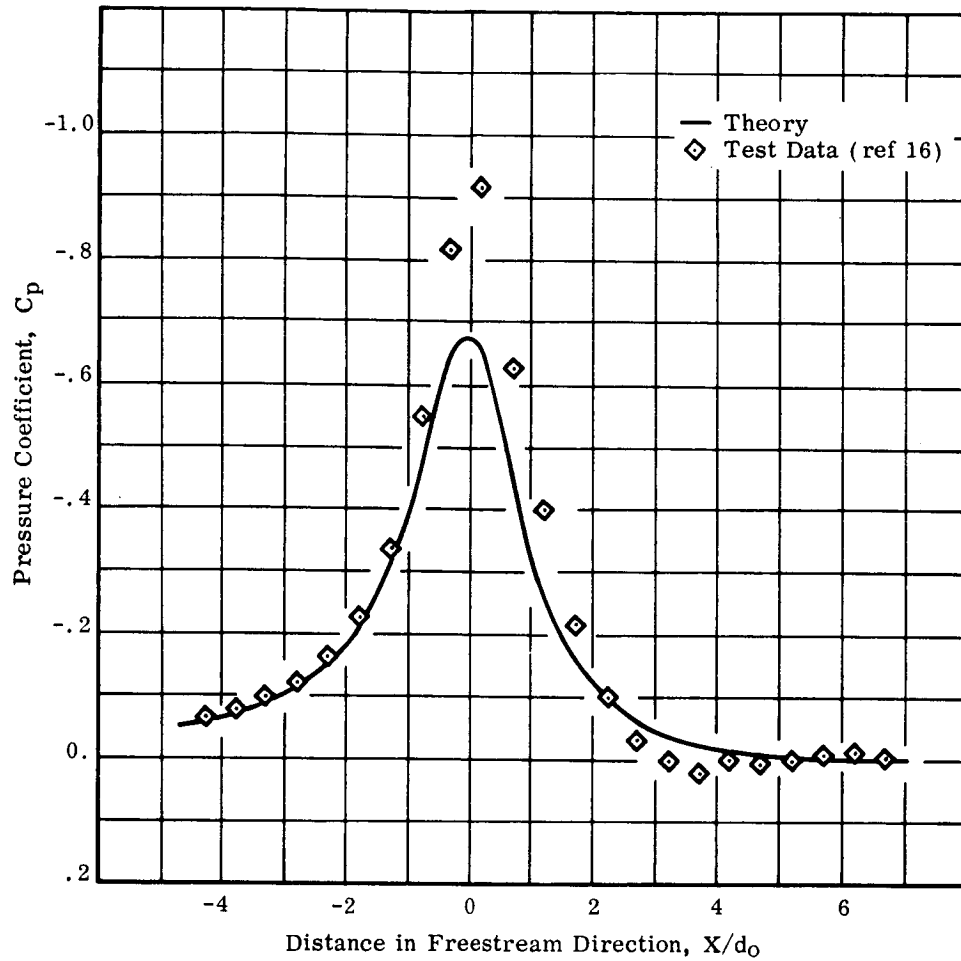
(a) Station  $Y/d_0 = 1.5$



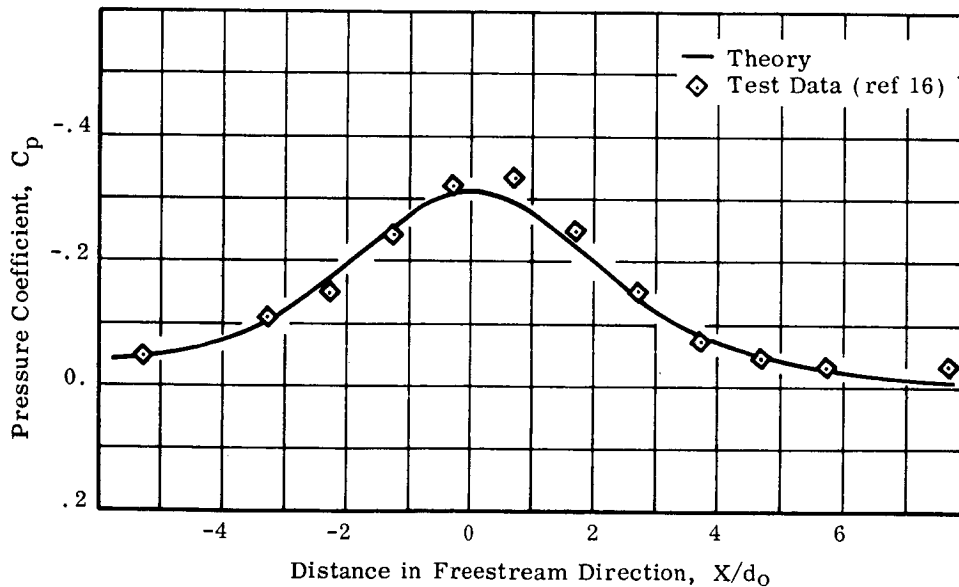
(b) Station  $Y/d_0 = 3$

FIGURE 8. INDUCED PRESSURE VARIATION FOR A VANED JET  
(Vaness Perpendicular to Freestream,  $U_\infty/U_{j0} = 0.125$ )



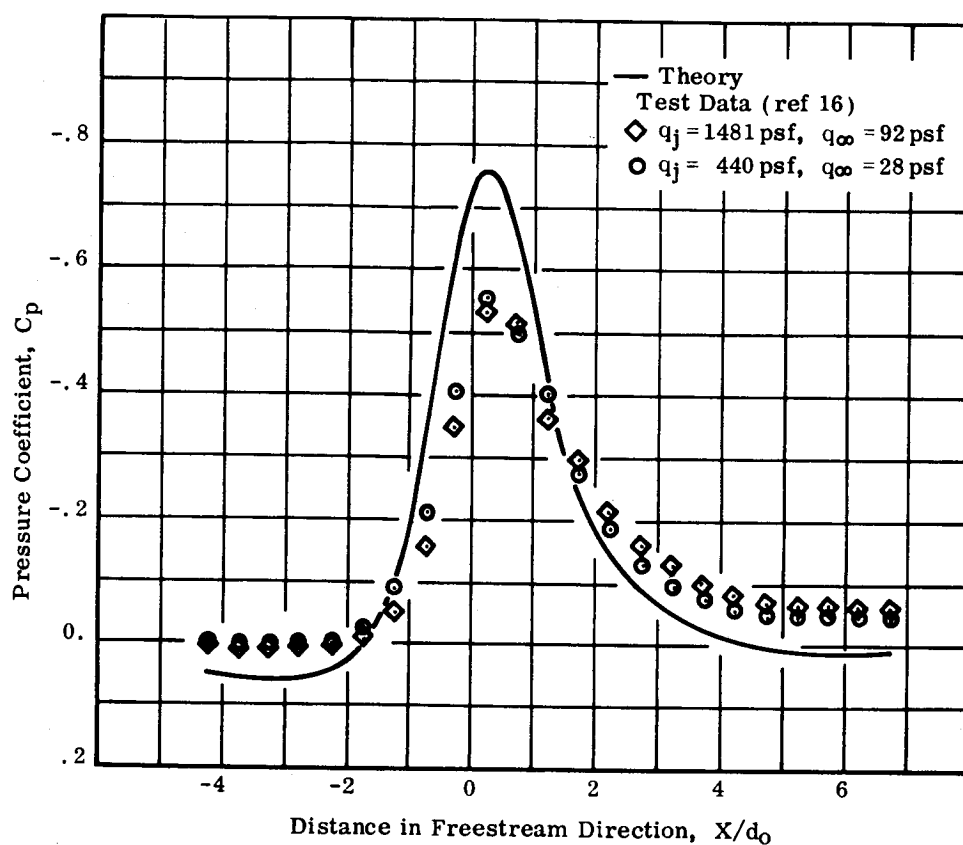


(a) Station  $Y/d_0 = 1.5$

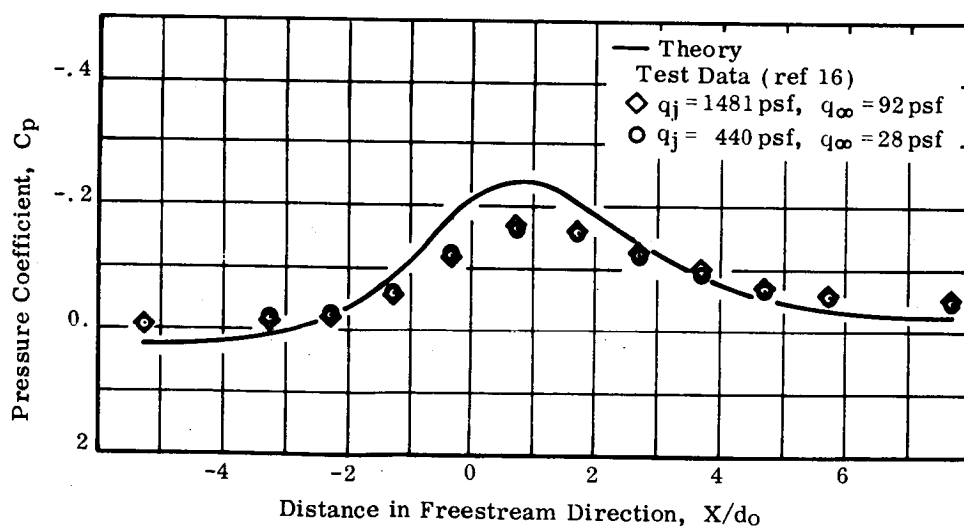


(b) Station  $Y/d_0 = 3$

FIGURE 9. INDUCED PRESSURE VARIATION FOR A VANED JET  
(Vaness Aligned with Freestream,  $U_\infty/U_{j0} = 0.125$ )

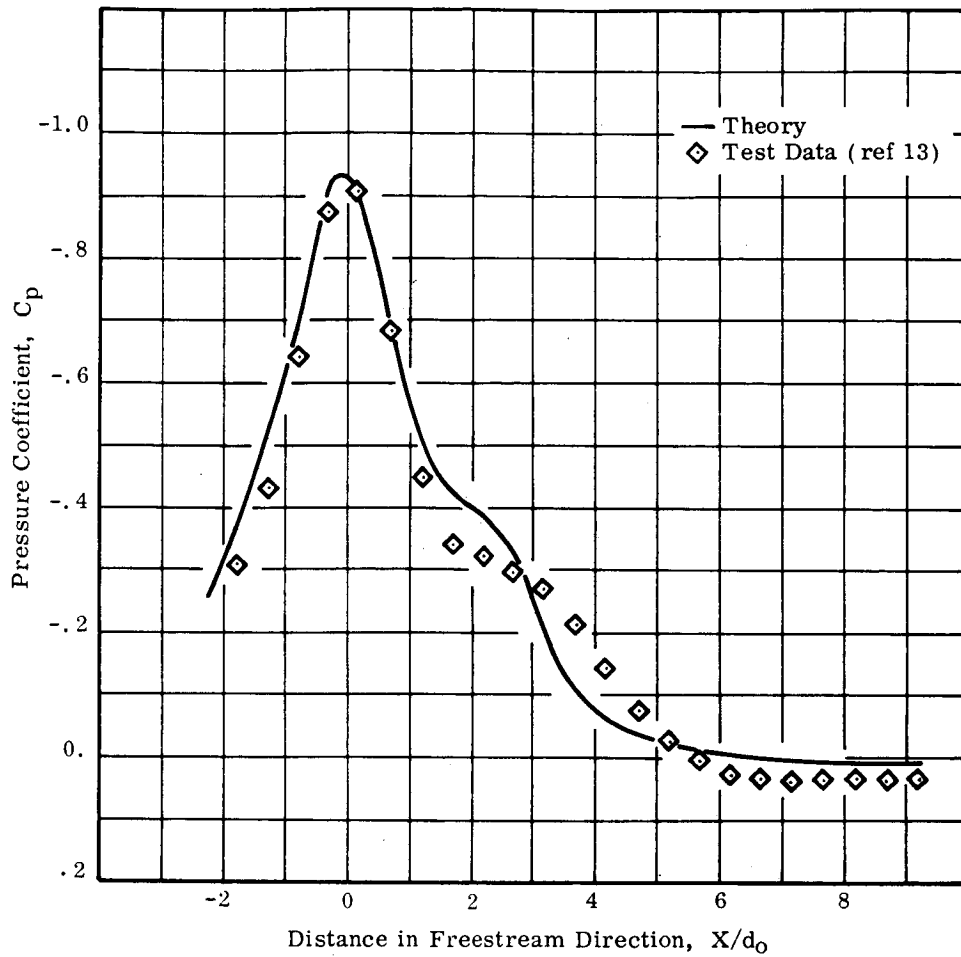


(a) Station  $Y/d_0 = 1.5$

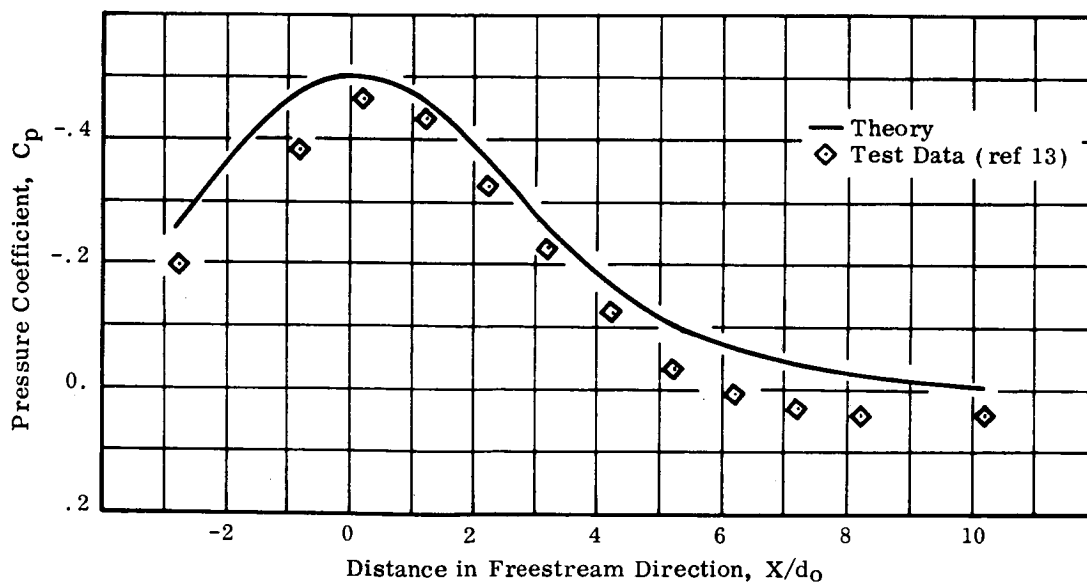


(b) Station  $Y/d_0 = 3$

FIGURE 10. INDUCED PRESSURE VARIATION FOR A VANED JET  
 (Vaness Perpendicular to Freestream,  $U_\infty/U_{j0} = 0.250$ )

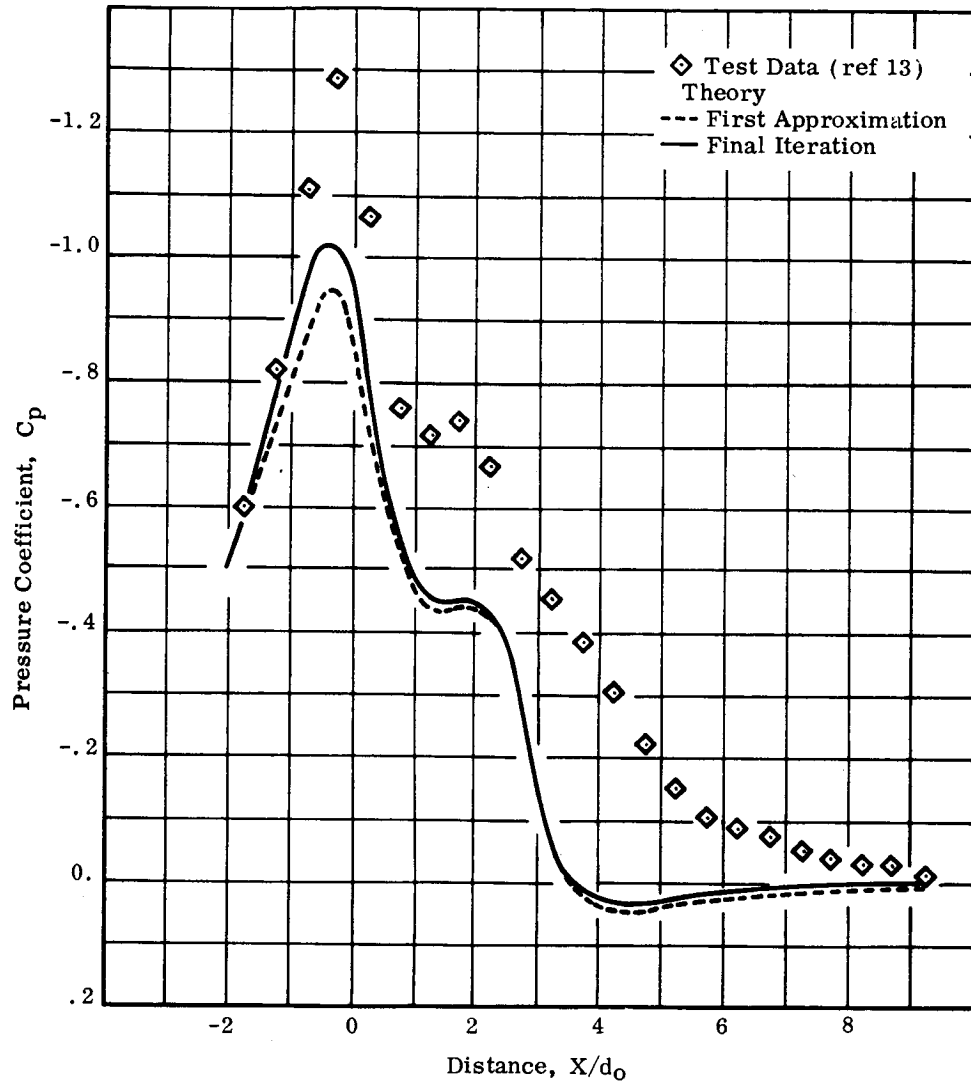


(a) Station  $Y/d_0 = 1.5$



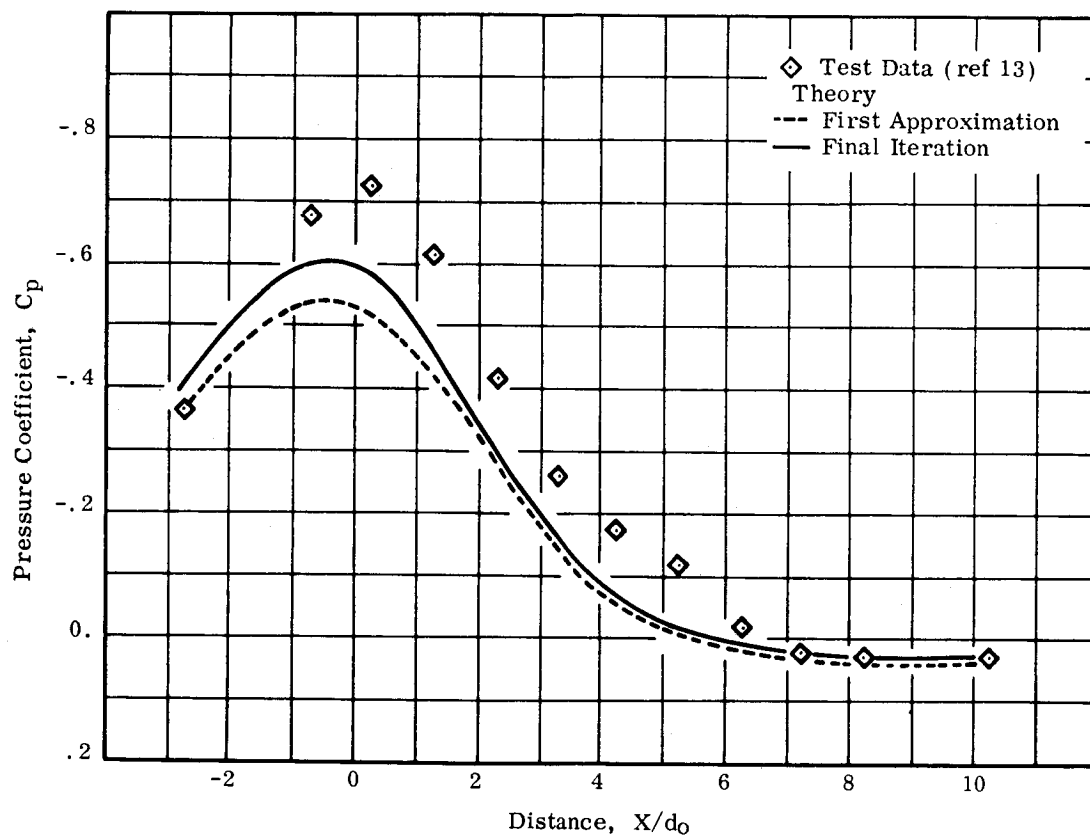
(b) Station  $Y/d_0 = 3$

FIGURE 11. INDUCED PRESSURE VARIATION FOR A TWO-JET CONFIGURATION AT ZERO SIDESLIP (Spacing =  $2.5 d_0$ ,  $\delta_j = 90^\circ$ ,  $U_\infty/U_{j0} = 0.125$ )



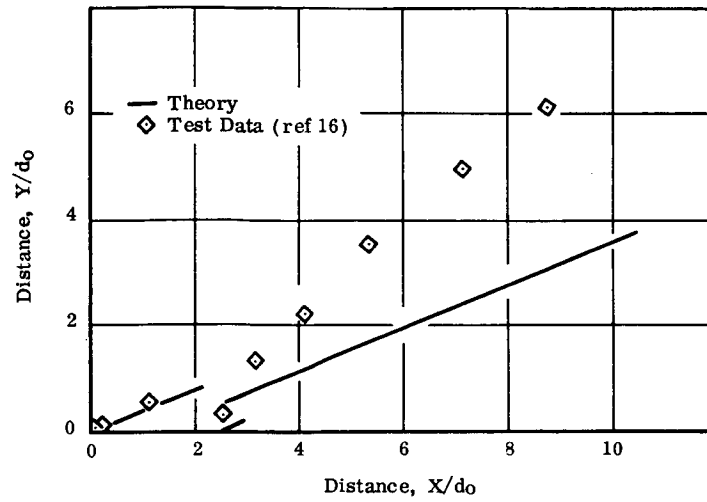
(a) Station  $Y/d_0 = 1.5$

FIGURE 12. INDUCED PRESSURE VARIATION FOR A TWO-JET CONFIGURATION  
AT SIDESLIP  $\beta = 20^\circ$  (Spacing =  $2.5 d_0$ ,  $\delta_j = 90^\circ$ ,  $U_\infty/U_{j0} = 0.125$ )

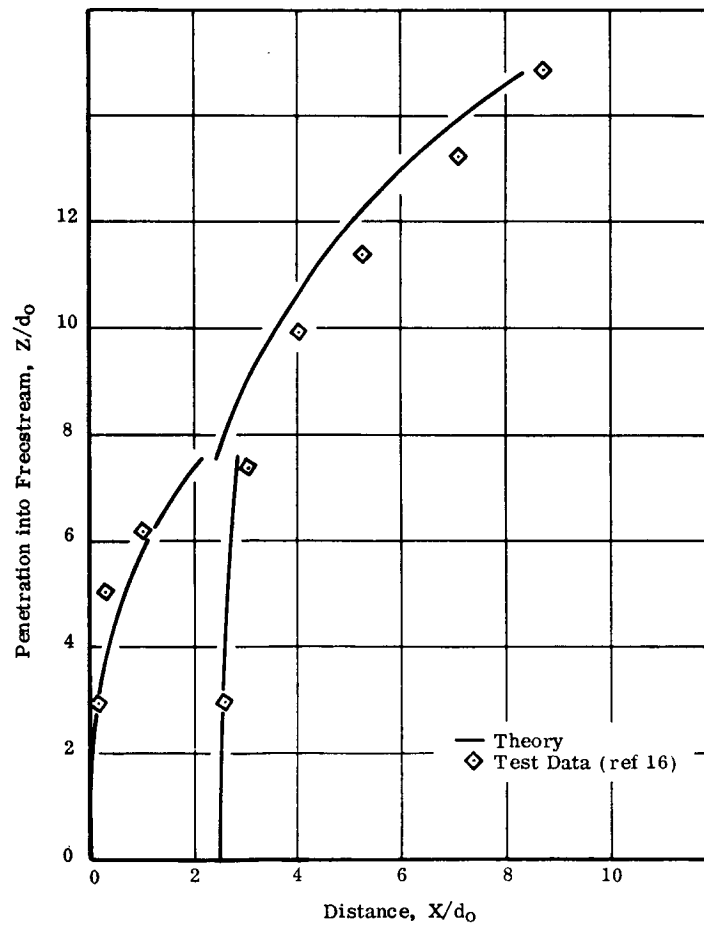


(b) Station  $Y/d_0 = 3$

FIGURE 12. (Concluded)

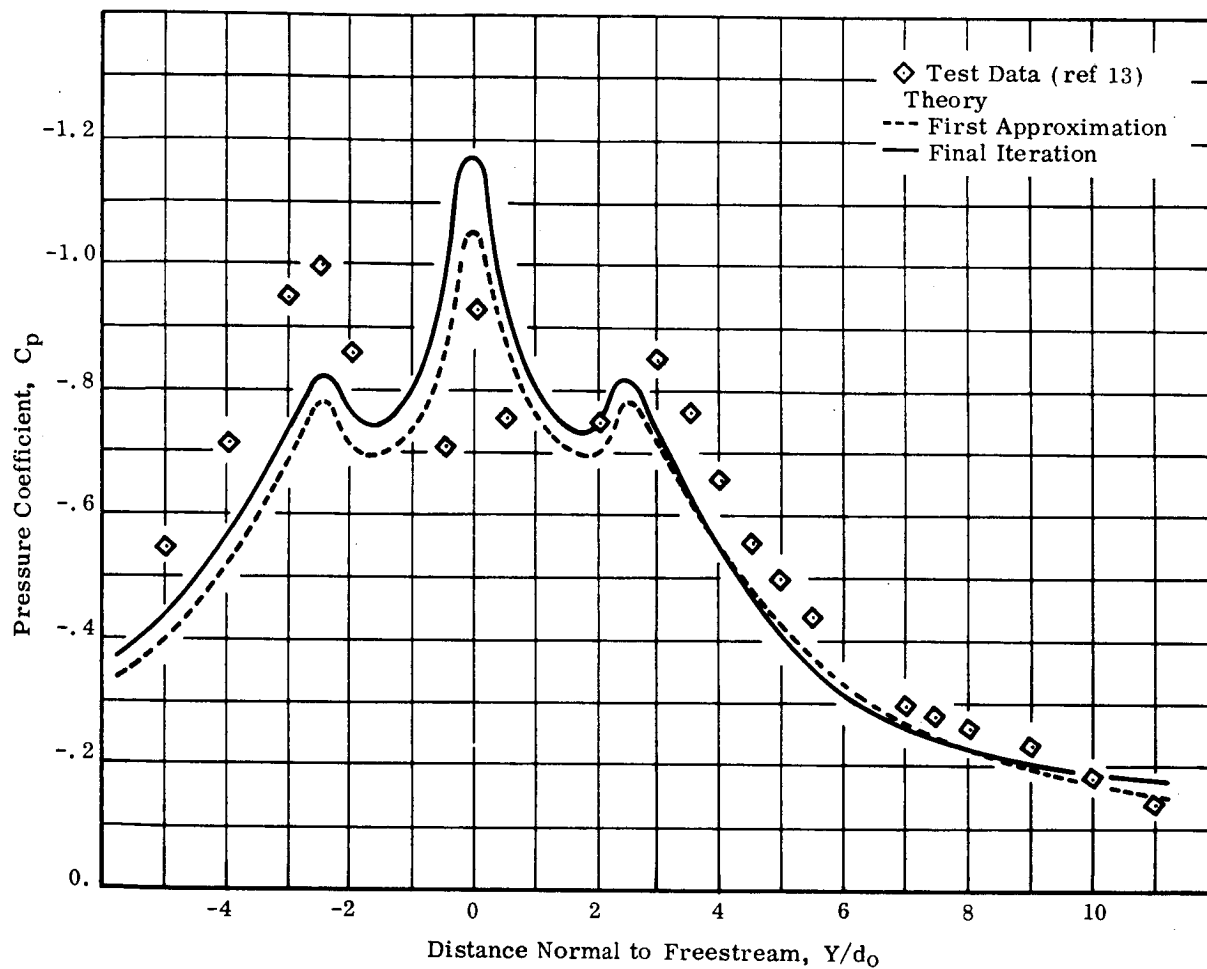


(a) Projection in X-Y Plane



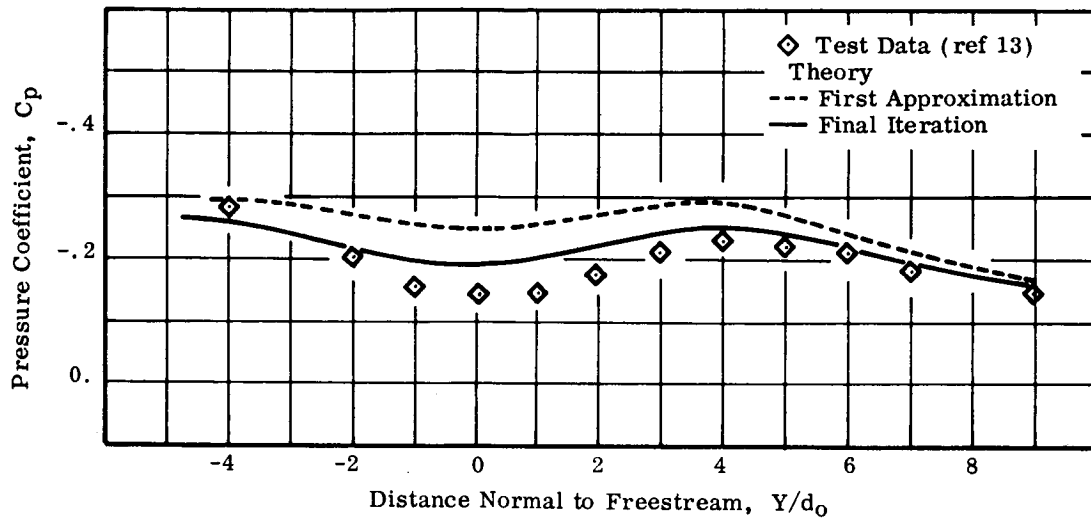
(b) Projection in X-Z Plane

FIGURE 13. CENTERLINES FOR A TWO-JET CONFIGURATION  
AT SIDESLIP  $\beta = 20^\circ$   
(Spacing =  $2.5 d_0$ ,  $\delta_j = 90^\circ$ ,  $U_\infty/U_{j0} = 0.125$ )

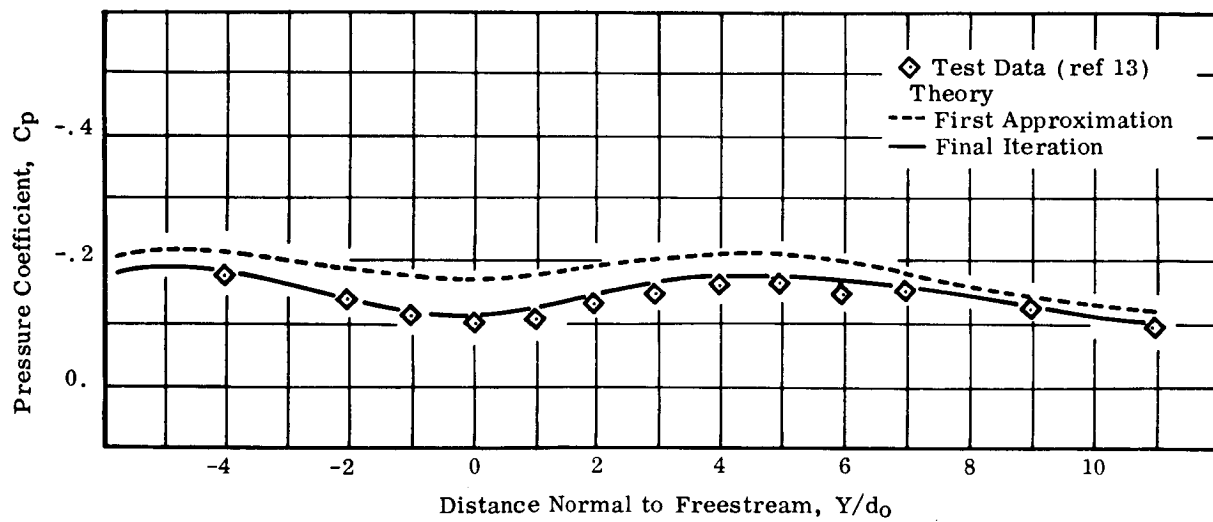


(a) Station  $X/d_0 = 0$

FIGURE 14. INDUCED PRESSURE VARIATION  
FOR A SPANWISE TWO-JET CONFIGURATION  
(Spacing =  $2.5 d_0$ ,  $\delta_j = 90^\circ$ ,  $U_\infty/U_{j0} = 0.125$ )



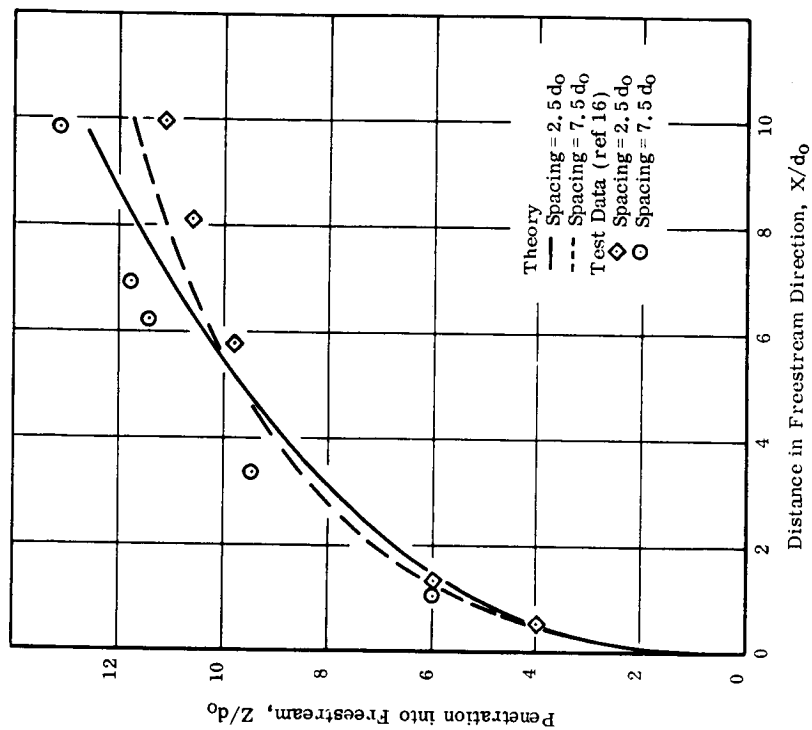
(b) Station  $X/d_0 = -3$



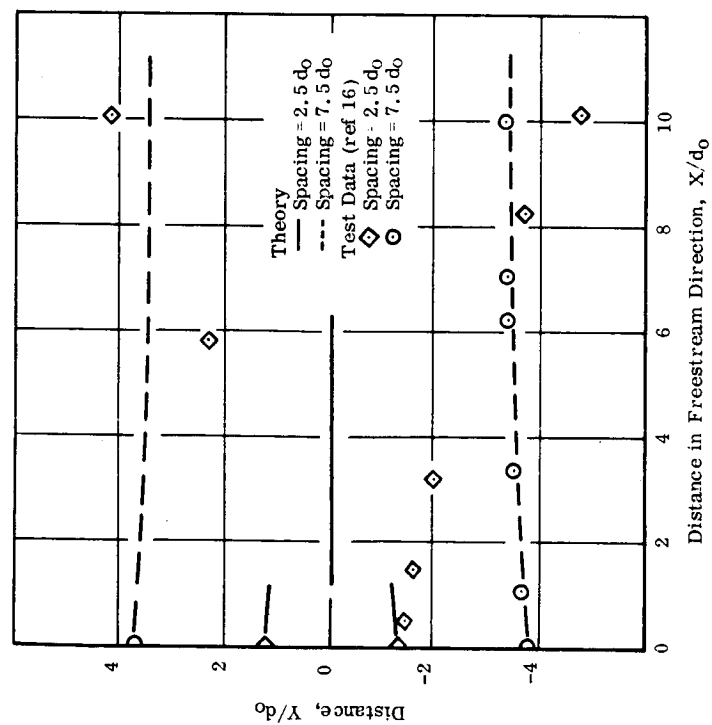
(c) Station  $X/d_0 = -4$

FIGURE 14. (Concluded)



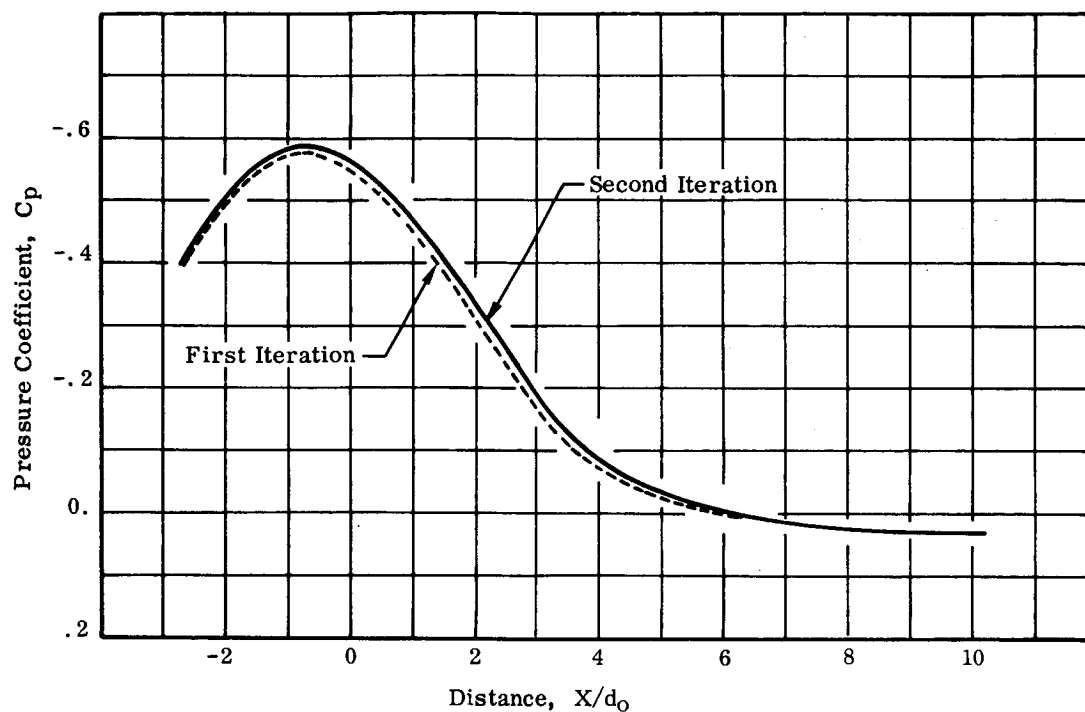


(a) Projection in X-Y Plane

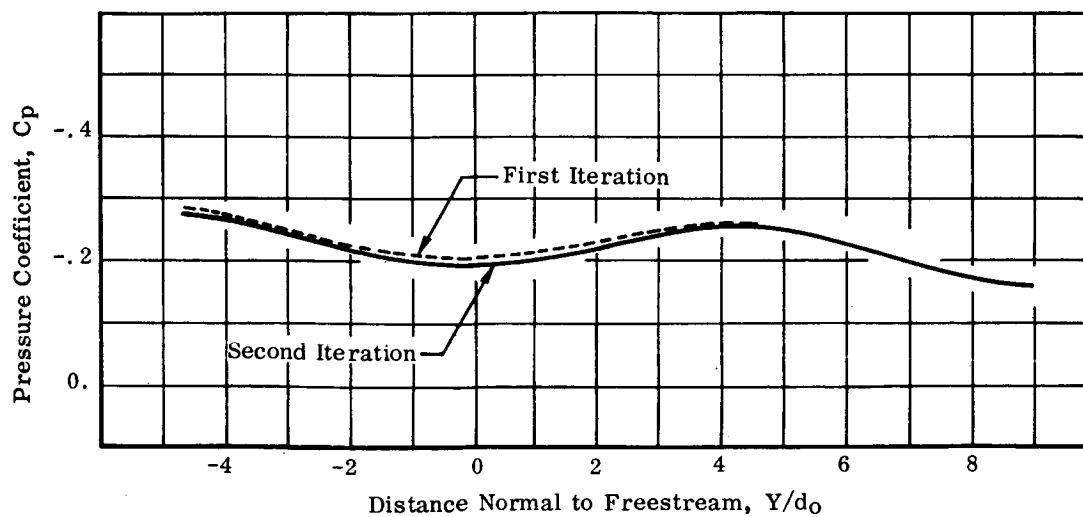


(b) Projection in X-Z Plane

FIGURE 15. CENTERLINES FOR SPANWISE TWO-JET CONFIGURATIONS  
 $(\delta_j = 90^\circ, U_\infty/U_{j0} = 0.125)$



(a) Iterations for Figure 12(b)



(b) Iterations for Figure 13(b)

FIGURE 16. CONVERGENCE OF COMPUTATIONAL METHOD

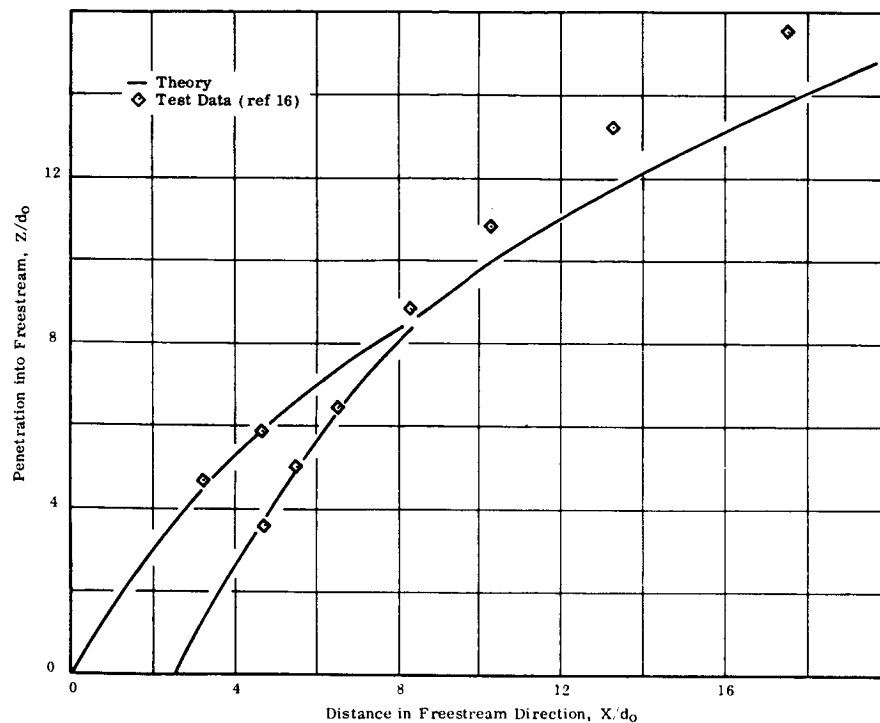


FIGURE 17. CENTERLINES FOR TWO INCLINED JETS  
(Spacing =  $2.5 d_o$ ,  $\delta_j = 60^\circ$ ,  $U_\infty/U_{j0} = 0.125$ )

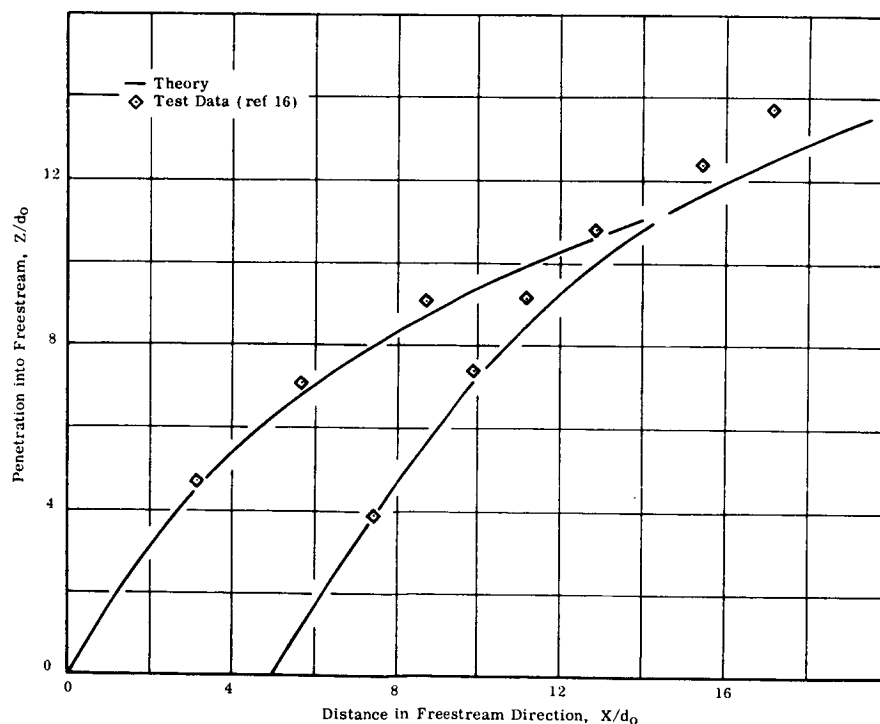


FIGURE 18. CENTERLINES FOR TWO INCLINED JETS  
(Spacing =  $5 d_o$ ,  $\delta_j = 60^\circ$ ,  $U_\infty/U_{j0} = 0.125$ )

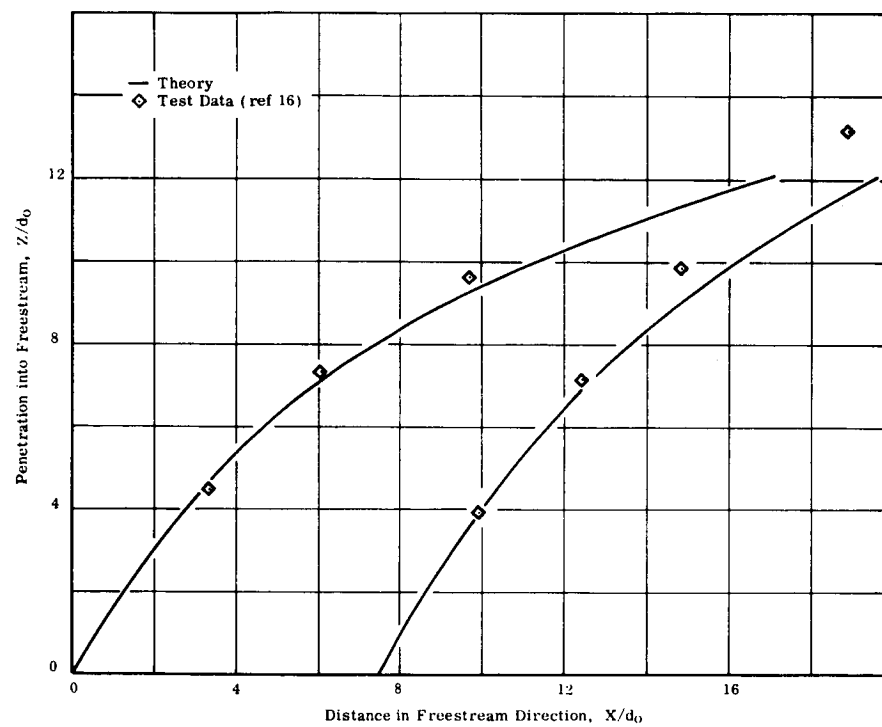
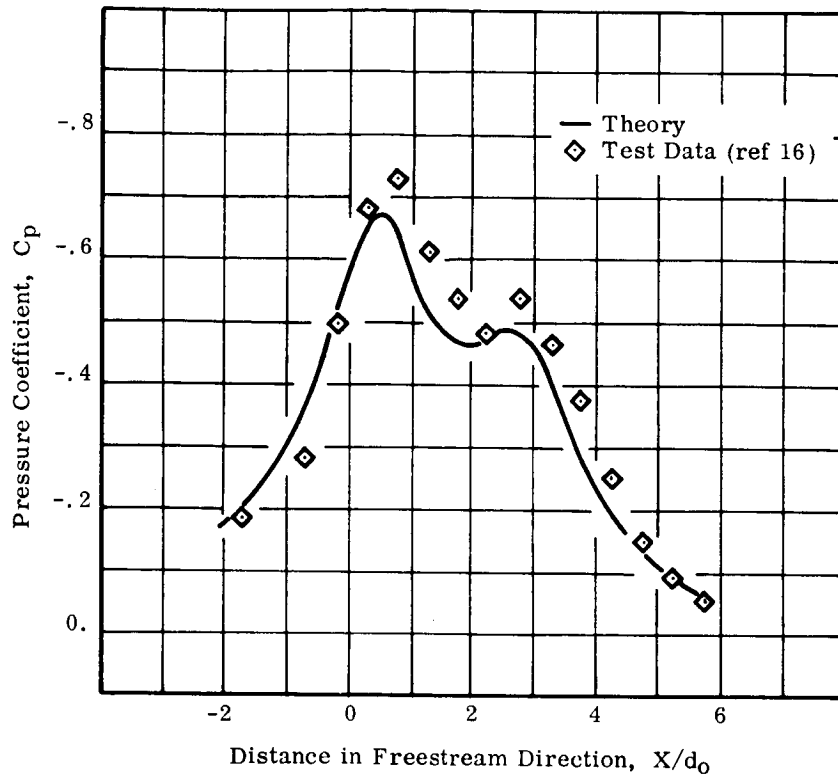
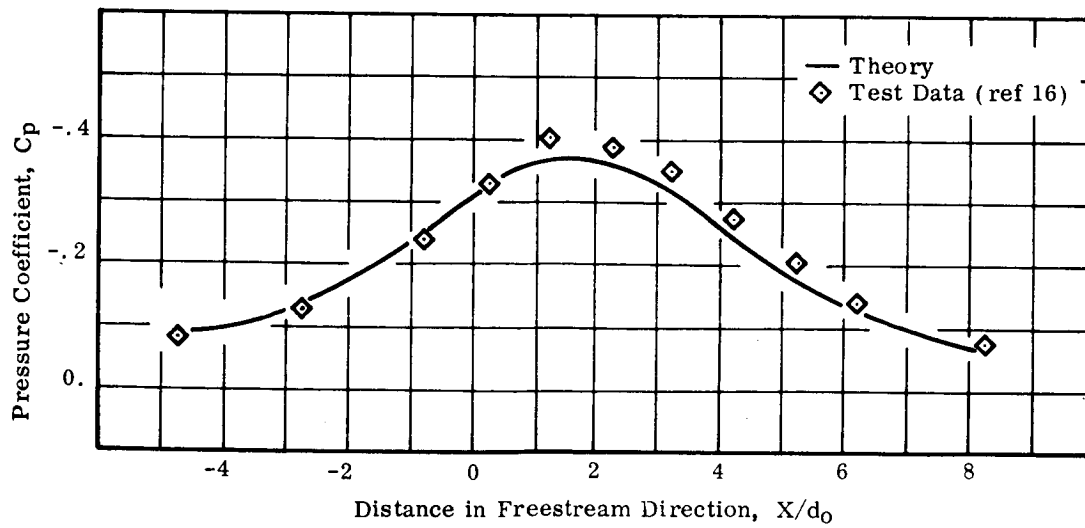


FIGURE 19. CENTERLINES FOR TWO INCLINED JETS  
(Spacing =  $7.5 d_0$ ,  $\delta_j = 60^\circ$ ,  $U_\infty/U_{j0} = 0.125$ )

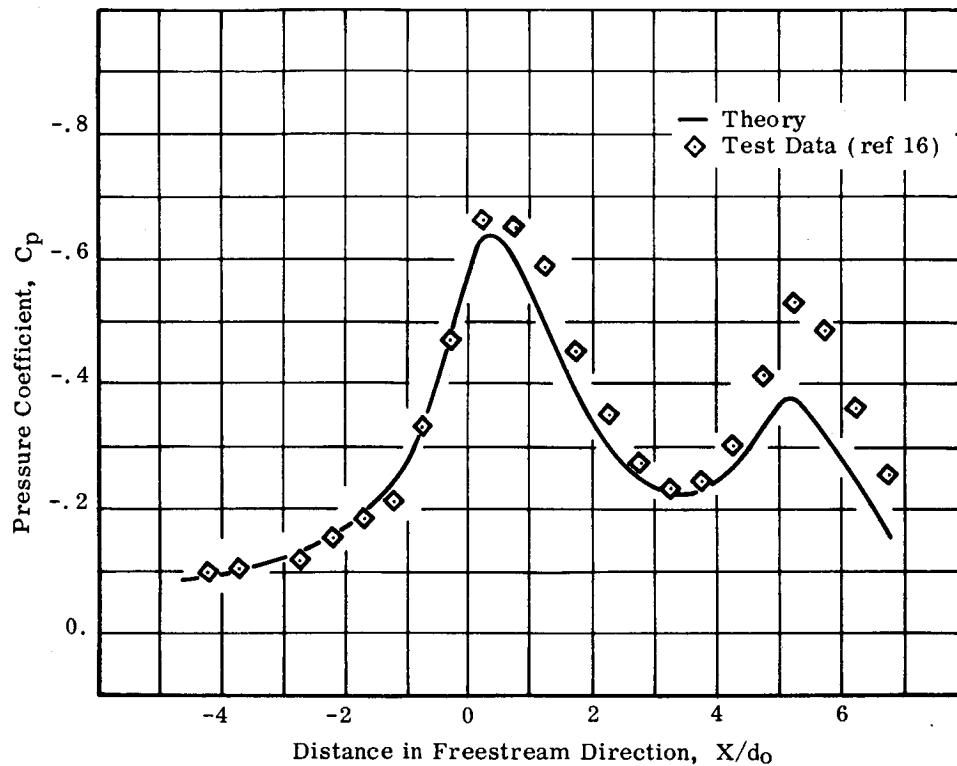


(a) Station  $Y/d_0 = 1.5$

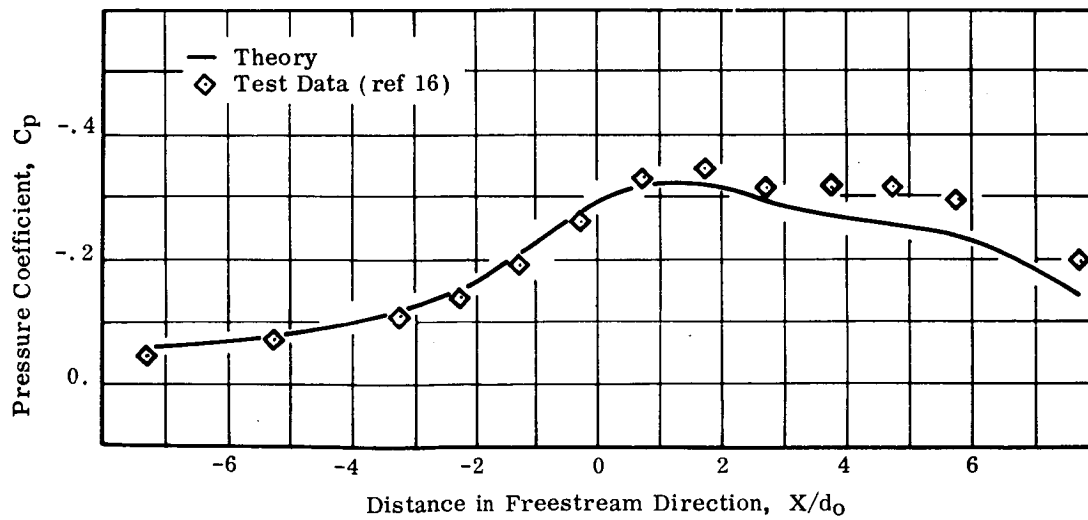


(b) Station  $Y/d_0 = 3$

FIGURE 20. INDUCED PRESSURE VARIATION FOR TWO INCLINED JETS  
(Spacing =  $2.5 d_0$ ,  $\delta_j = 60^\circ$ ,  $U_\infty/U_{j0} = 0.125$ )

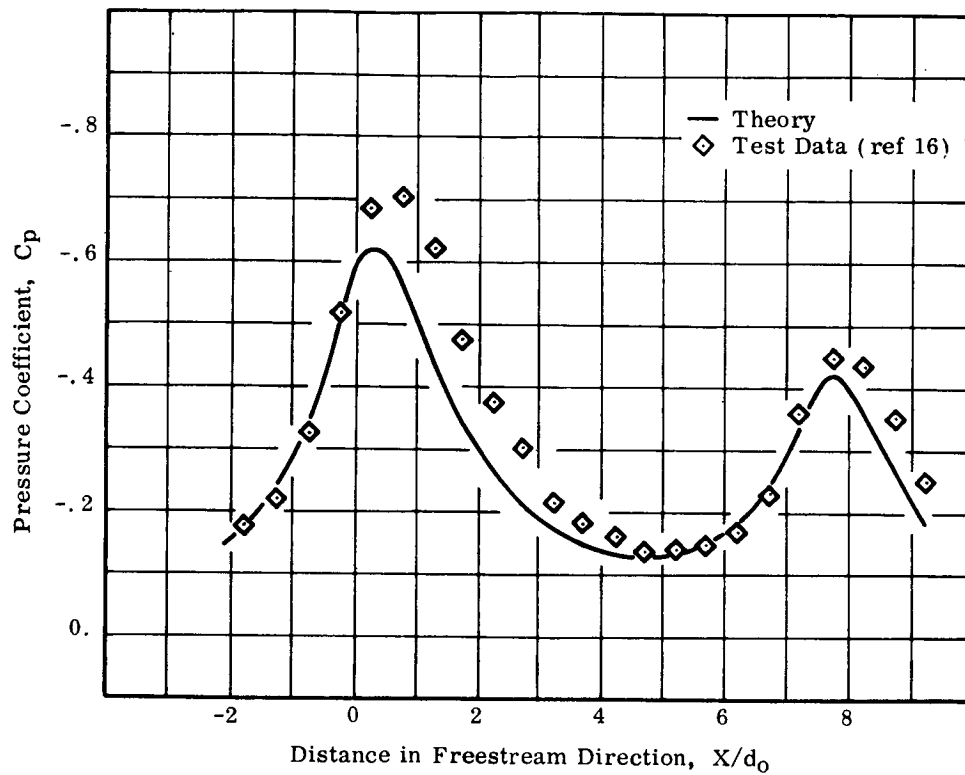


(a) Station  $Y/d_0 = 1.5$

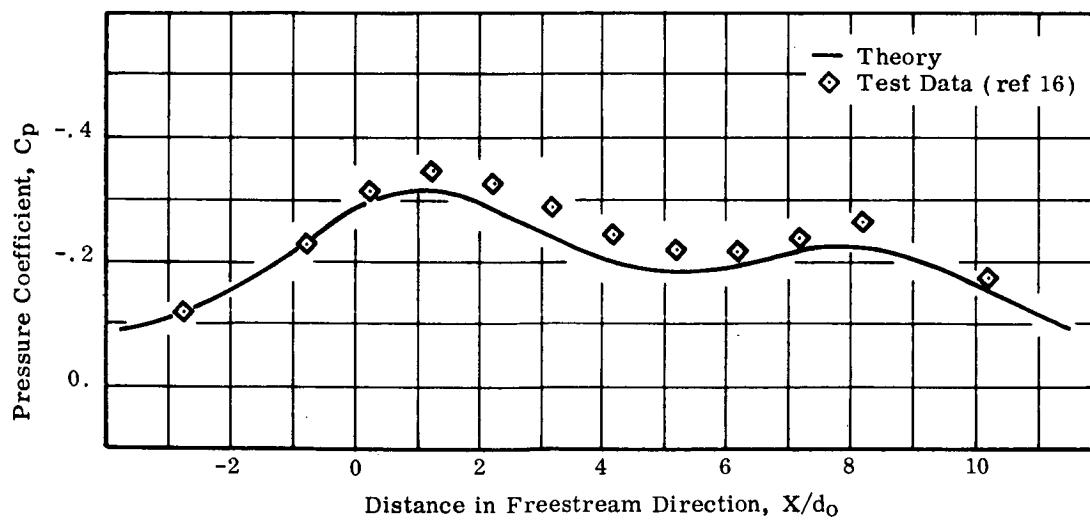


(b) Station  $Y/d_0 = 3$

FIGURE 21. INDUCED PRESSURE VARIATION FOR TWO INCLINED JETS  
(Spacing =  $5 d_0$ ,  $\delta_j = 60^\circ$ ,  $U_\infty/U_{j0} = 0.125$ )

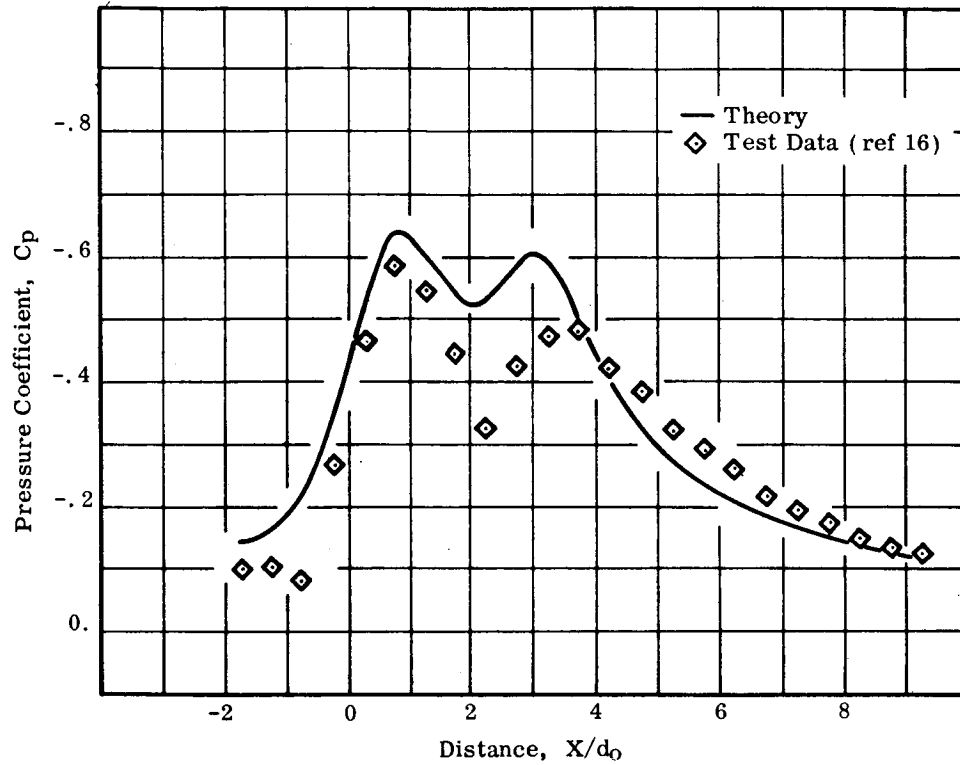


(a) Station  $Y/d_0 = 1.5$

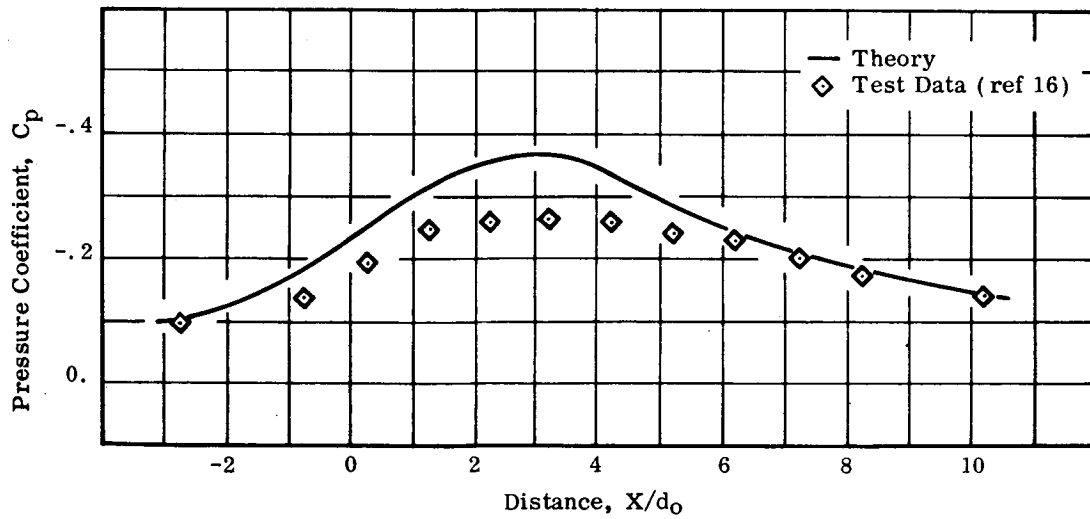


(b) Station  $Y/d_0 = 3$

FIGURE 22. INDUCED PRESSURE VARIATION FOR TWO INCLINED JETS  
(Spacing =  $7.5 d_0$ ,  $\delta_j = 60^\circ$ ,  $U_\infty/U_{j0} = 0.125$ )



(a) Station  $Y/d_0 = 1.5$



(b) Station  $Y/d_0 = 3$

FIGURE 23. INDUCED PRESSURE VARIATION FOR TWO INCLINED JETS  
AT SIDESLIP  $\beta = 20^\circ$  (Spacing =  $2.5 d_0$ ,  $\delta_j = 60^\circ$ ,  $U_\infty/U_{j0} = 0.125$ )



## APPENDIX A

### APPLICATION OF COMPUTER PROGRAM

#### Sample Problem

The use of the computer program in determining jet deflections and the jet-induced flow field is demonstrated for a general two-jet configuration exhausting into a crossflow.

The two-jet configuration used in this sample problem is not one of the configurations considered in the experimental phase of this study. Rather, it is a composite configuration designed to exercise the computer program in its most general mode. This configuration is shown in figure A1. Nozzle #1 is the annular nozzle with dead air core described on pages 10-13. Nozzle #2 has the exit flow characteristics of the vaned nozzle described on pages 13 and 14, but has an exit diameter twice as large. Differing jet exit velocity ratios have been used for the two jets in the configuration.

#### Input Data for Sample Problem

The input data cards required for the sample problem are tabulated in figure A2 and are described below.

Card 1 lists five control indices. The first one, MULT=2, indicates that a two-jet configuration is being treated. The second one, IGEØM=4, specifies that pressure coefficients, as well as induced velocity components, are to be evaluated at all the control points provided as part of the input. By setting IPUNCH=0, no punched output is generated. The next two control indices deal with the calculation of mutual interference effects, as discussed on pages 16-20. NPS=10 specifies that there will be 10 integration intervals in each segment of each exhausting jet for which a modified freestream velocity vector will be computed. After establishing an initial approximation using the unperturbed crossflow velocity vector, two iterations, utilizing crossflow velocity vectors modified by mutually induced velocities for each segment, are specified by ITER=2.

Card 2 specifies the angle of attack,  $\alpha = 0$ , and angle of sideslip,  $\beta = 20^\circ$ , in the coordinate system of figure A1.

Card 3 controls the number of intervals and the interval size in the numerical integration of the equations of motion for the jet path. The integration is carried out in a local, jet-oriented coordinate system defined by the freestream velocity vector and the initial jet exhaust vector (for details, see reference 12). The numerical integration routine in the program will optimize the actual integration step size utilized. Data for the jet centerline will be printed out at the specified intervals. For the sample problem, 80 intervals and an interval size of 0.5 jet exit diameters are chosen.

Cards 4, 5 and 6 describe the upstream jet. The jet location, in the coordinate system of figure A1, is  $X=-3.75$ ,  $Y=0.0$ ,  $Z=0.0$ . The jet exhaust angles  $\phi$  and  $\psi$  are  $180^\circ$  and  $0$ , respectively. The jet exit diameter,  $d_o=1.0$ , and the jet exit velocity ratio,  $U_\infty/U_{j0}=0.125$ , are given. The parameters on card 6 serve to describe the exit flow characteristics of the nozzle. The ratio of effective core diameter to jet exit diameter for the annular nozzle with dead air core is 0.40 and the jet mixing parameter is 0.10 (see discussion, page 13).

Cards 7, 8 and 9 describe the downstream jet, which is located at  $X=-1.25$ ,  $Y=0.0$ ,  $Z=0.0$ , and again exhausts normally into the crossflow. The jet exit diameter is 2.0 and the jet exit velocity ratio is 0.250. The stratified exit flow characteristics of the vaned nozzle are specified by the parameters 0.58 and 0.10 (see page 14).

Card 10 lists the parameter controlling the initial cross section of the jet resulting from the coalescence of the two exhausting jets. An ellipse with a minor to major axis ratio of 0.5 is specified. (See Appendix B for guidelines on this parameter).

Card 11 lists the number of spanwise stations,  $NS=3$ , and the number of control points at each station,  $NC=4$ , where the induced flow properties are to be evaluated.

Cards 12-17 list the coordinates of the control points. All lie in the plane of the jet exits. The coordinates for each control point appear in the order  $X$ ,  $Y$ ,  $Z$ . The total number of control points is  $NC \times NS$ . The listing is continuous, i.e., no new record is required for the start of each spanwise station.

#### Output for Sample Problem

For the option specified on card 1, only printed output is obtained. Figure A3(a) shows the first page of printed output. The jet configuration being treated is identified both by appropriate heading as well as other pertinent input data. Input controlling the numerical integration procedure is also displayed.

Figure A3(b) shows the jet centerline computations for the initial approximation (i. e., the crossflow velocity vector is unperturbed). The coordinates of the jet centerline, the nondimensionalized mean jet speed,  $U_j/U_{j0}$ , and the nondimensionalized major diameter of the ellipse representing the cross section of the jet,  $d/d_0$ , are printed out for each exhausting jet up to the point of coalescence. The point of coalescence of the two jets is identified and initial conditions for the resulting jet are printed out. Jet centerline information for this jet, resulting from the intersection of the two exhausting jets, is then displayed. Jet centerline data are printed out at each integration interval specified on card 3 of the input data, since for a normally exhausting jet at zero angle of attack the local jet-oriented coordinate system in which integration is carried out and the fixed input/output coordinate system coincide. The output in figure A3(b) displays only a portion of the jet centerline data generated for the sample problem. Computations would, of course, extend to  $Z = -40.0$ , which represents integration of the equations of motion over the range  $|Z| = 80 \times 0.5 \times 1.0 = 40.0$ , as specified on card 3 of the input data.

Figure A3(d) shows the printout for the jet-induced velocity components and pressure coefficients at the control points specified. The coordinates of the control points are identified. The pressure coefficients and the induced velocity components  $U$ ,  $V$ ,  $W$ , nondimensionalized by  $U_\infty$ , are given.

The printout of figures A3(b) and A3(d) would then be repeated for the number of iterations specified on card 1. Figures A3(c) and A3(e) show the computations for the second, or final, iteration for the sample problem. Note that now, with the crossflow velocity vector modified by mutually induced velocities for each segment, each jet-oriented coordinate system is no longer aligned with the fixed input/output coordinate system. Thus, printout no longer occurs at the regular intervals of 0.5, 1.0, 1.5, etc. The program does adjust the local-coordinate integration interval to maintain consistently spaced print on jet centerline data.

### Applicability and Limitations

The program may be utilized to evaluate the induced flow field due to one or two jets exhausting into a crossflow. Jet exit velocity stratification effects may be treated by employing the velocity ratio and jet exit diameter for the equivalent ideal nozzle, or, alternatively, by accounting for the internal mixing through the introduction of the parameters describing jet exit flow characteristics.

For a single-jet configuration, the initial jet exhaust direction, specified by  $\phi$

and  $\psi$ , and the freestream direction, specified by  $\alpha$  and  $\beta$ , may be arbitrary. For a two-jet configuration, the jet exits must both lie in the same XY plane and the jet exhaust planes, defined by the freestream vector and the initial jet exhaust vectors, must be parallel.

Comparisons between computations and experimental data have been made for velocity ratios  $0.10 \leq U_\infty/U_{j0} \leq 0.30$ , and the program may be considered most applicable in this range.

The choice of variables governing the numerical integration for the jet path is related to the velocity ratio of the problem being considered. For  $U_\infty/U_{j0} \leq 0.125$ , integration in the direction normal to the freestream over an extent of at least 30 jet exit diameters has been found desirable. As  $U_\infty/U_{j0}$  increases, this may be decreased, as the jet penetrates less at the higher velocity ratios. For the above range of velocity ratios an integration interval size of  $\leq 0.5$  jet exit diameters has been found satisfactory.

Control points at which jet-induced properties are to be evaluated may not lie within the jet itself, as the theory is not valid in this region. Generally, control points positioned less than 2 jet exit diameters from the center of a jet exit should be avoided, to minimize distortion in the computed velocity distributions.

The jet-induced velocity field may be employed to explore the interaction between exhausting jets and adjacent supporting structures. Loading on adjacent lifting surfaces may be evaluated by lifting surface theory, and other techniques, such as the transformation method, may be utilized for this purpose on fuselages. Details on the application of these methods may be found in reference 12, where the treatment of more complex multiple-jet configurations is also discussed.

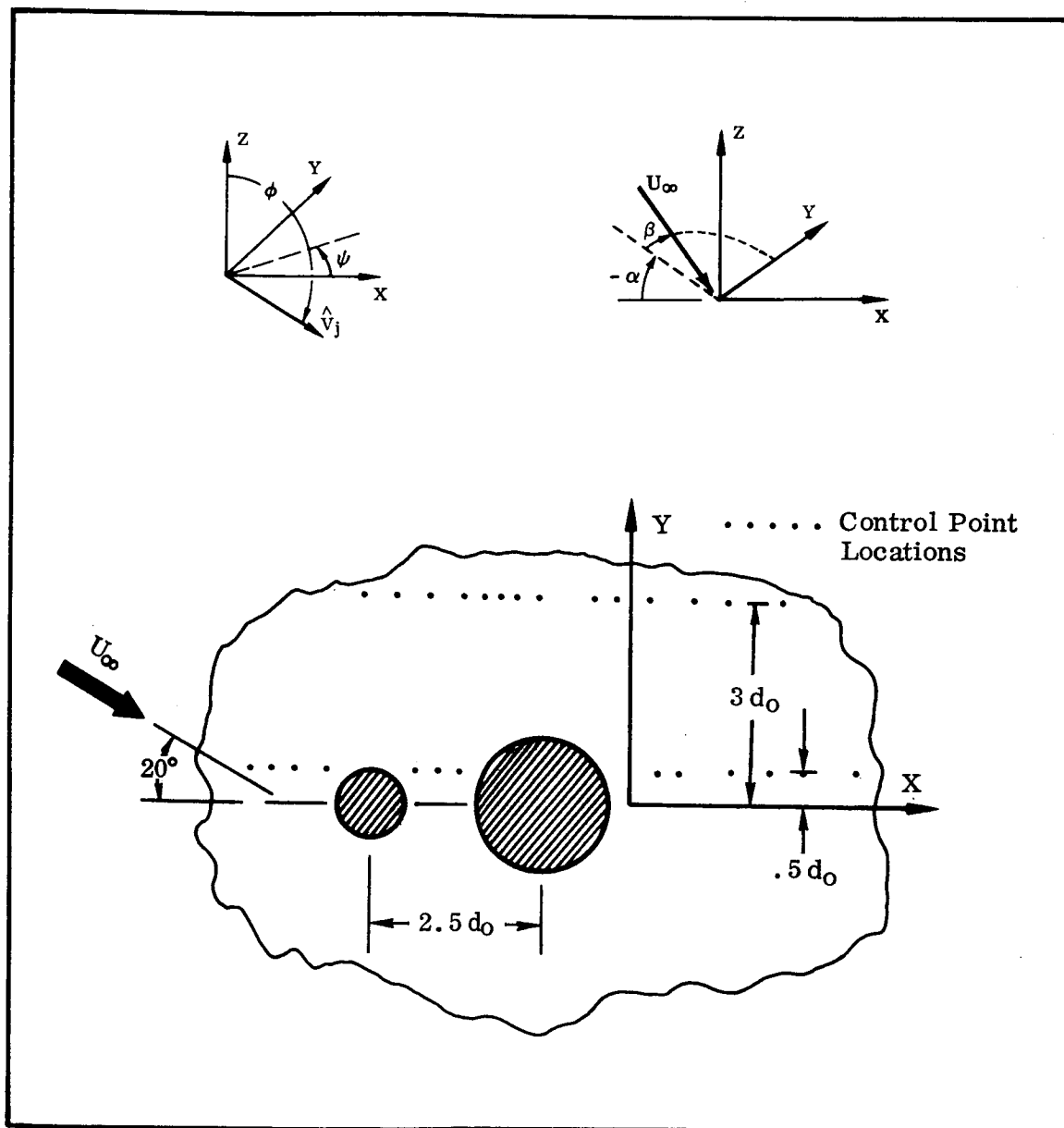


FIGURE A1. JET-CONFIGURATION OF SAMPLE PROBLEM

Diagram illustrating a sequence of nodes (1 through 12) followed by an arrow pointing to node 17.

FIGURE A2. INPUT DATA FOR SAMPLE PROBLEM

```

*** TWO-JET CONFIGURATION ***

XJET      YJET      ZJET      PHI      PSI      U/UJ0      D0
-3.7500    0.0000    0.0000    180.0000    0.0000    .1250    1.0000
-1.2500    0.0000    0.0000    180.0000    0.0000    .2500    2.0000

NONUNIFORM EXIT FLOW PARAMETERS FOR NOZZLE OF JET 1:  A1 = .400  B1 = .100
NONUNIFORM EXIT FLOW PARAMETERS FOR NOZZLE OF JET 2:  A2 = .580  B2 = .100

ANGLE OF ATTACK = 0.00
ANGLE OF SIDESLIP = 20.00

NUMBER OF STEPS IN INTEGRATION = 80
INTEGRATION INTERVAL = .50 JET EXIT DIAMETERS

INITIAL RATIO OF MINOR TO MAJOR DIAMETER OF ELLIPTICAL CROSS SECTION OF COALESCED JET IS .5

```

(a) Run Identification Parameters

FIGURE A3. OUTPUT FOR SAMPLE PROBLEM

\*\*\* INITIAL APPROXIMATION \*\*\*

\*\* CENTERLINES OF JETS 1 AND 2 \*\*  
AND COALESCED JET

\*\*\*\*\*

XCOORD	YCOORD	ZCOORD	UJ	DIA	XCOORD	YCOORD	ZCOORD	UJ	DIA
-3.75	0.00	0.00	1.000	1.00	-1.25	0.00	0.00	1.000	1.00
-3.75	-0.00	-0.50	.946	1.12	-1.24	-0.00	-0.50	.951	1.09
-3.74	-0.00	-1.00	.893	1.29	-1.23	-0.01	-1.00	.905	1.21
-3.72	-0.01	-1.50	.840	1.54	-1.20	-0.02	-1.50	.861	1.36
-3.69	-0.02	-2.00	.783	1.94	-1.16	-0.03	-2.00	.819	1.54
-3.64	-0.04	-2.50	.714	2.54	-1.11	-0.05	-2.50	.780	1.78
-3.57	-0.06	-3.00	.645	2.85	-1.04	-0.08	-3.00	.714	2.09
-3.48	-0.10	-3.50	.586	3.14	-.96	-0.11	-3.50	.716	2.54
-3.36	-0.14	-4.00	.535	3.44	-.86	-0.14	-4.00	.663	2.79
-3.22	-0.19	-4.50	.491	3.77	-.73	-0.19	-4.50	.614	2.95
-3.04	-0.26	-5.00	.452	4.11	-.59	-0.24	-5.00	.573	3.11
-2.83	-0.33	-5.50	.419	4.47	-.42	-0.30	-5.50	.538	3.28
-2.58	-.43	-6.00	.389	4.84	-.23	-.37	-6.00	.509	3.45
-2.29	-.53	-6.50	.362	5.23	-.01	-.45	-6.50	.504	3.63
-1.95	-.65	-7.00	.339	5.64	.23	-.54	-7.00	.480	3.81
-1.57	-.79	-7.50	.319	6.07	.49	-.64	-7.50	.459	3.99
-1.12	-.96	-8.00	.300	6.50	.79	-.74	-8.00	.440	4.18
-.62	-1.14	-8.50	.284	6.96	1.11	-.86	-8.50	.422	4.38
-.04	-1.35	-9.00	.270	7.42	1.45	-.98	-9.00	.405	4.57
.62	-1.59	-9.50	.258	7.90	1.83	-1.12	-9.50	.390	4.77
1.37	-1.86	-10.00	.246	8.40	2.23	-1.27	-10.00	.376	4.98
2.21	-2.17	-10.50	.236	8.91	2.67	-1.43	-10.50	.363	5.19

PROPERTIES OF COALESCED JET X= 2.44 Y= -1.80 Z= -10.50 U/UJ0= .61 D= 9.65

XCOORD	YCOORD	ZCOORD	UJ	DIA
2.44	-1.80	-10.50	1.000	1.00
3.12	-2.04	-11.00	.982	1.08
3.85	-2.31	-11.50	.964	1.17
4.65	-2.60	-12.00	.946	1.28
5.53	-2.92	-12.50	.928	1.42
6.50	-3.28	-13.00	.911	1.58
7.58	-3.67	-13.50	.894	1.70
8.78	-4.11	-14.00	.877	1.76
10.11	-4.59	-14.50	.863	1.82
11.60	-5.13	-15.00	.849	1.88
13.24	-5.73	-15.50	.836	1.94
15.06	-6.39	-16.00	.825	2.00
17.08	-7.13	-16.50	.814	2.07
19.32	-7.94	-17.00	.804	2.13
21.79	-8.84	-17.50	.794	2.19
24.53	-9.84	-18.00	.785	2.26
27.56	-10.94	-18.50	.777	2.33
30.91	-12.16	-19.00	.768	2.39
34.62	-13.51	-19.50	.761	2.46
38.71	-15.00	-20.00	.754	2.53

(b) Jet Centerlines (Initial Approximation)

FIGURE A3. (Continued)



\*\*\* ITERATION NUMBER 2, FINAL ITERATION \*\*\*

\*\* CENTERLINES OF JETS 1 AND 2 \*\*  
AND COALESCED JET

\*\*\*\*\*

XCOORD	YCOORD	ZCOORD	UJ	DIA	XCOORD	YCOORD	ZCOORD	UJ	DIA
-3.75	0.00	0.00	1.000	1.00	-1.25	0.00	0.00	1.000	1.00
-3.75	-.00	-.50	.945	1.12	-1.25	-.00	-.50	.953	1.09
-3.74	-.01	-1.00	.892	1.24	-1.23	-.01	-1.00	.908	1.20
-3.72	-.01	-1.50	.839	1.54	-1.21	-.02	-1.50	.867	1.34
-3.69	-.03	-2.00	.781	1.95	-1.17	-.03	-2.01	.829	1.50
-3.64	-.04	-2.49	.711	2.59	-1.12	-.05	-2.51	.794	1.69
-3.57	-.07	-2.99	.642	2.86	-1.06	-.08	-3.02	.760	1.95
-3.48	-.11	-3.49	.583	3.15	-.99	-.10	-3.52	.724	2.32
-3.37	-.15	-3.98	.532	3.46	-.90	-.14	-4.03	.685	2.74
-3.22	-.21	-4.47	.488	3.79	-.80	-.18	-4.54	.646	2.88
-3.05	-.28	-4.96	.449	4.13	-.68	-.23	-5.04	.615	3.01
-2.84	-.37	-5.46	.416	4.48	-.55	-.30	-5.55	.587	3.15
-2.60	-.48	-5.96	.387	4.84	-.40	-.37	-6.06	.561	3.29
-2.32	-.61	-6.46	.362	5.22	-.25	-.45	-6.57	.537	3.44
-2.02	-.77	-6.97	.339	5.62	-.08	-.55	-7.08	.515	3.59
-1.67	-.95	-7.47	.319	6.03	.11	-.65	-7.60	.494	3.74
-1.27	-1.17	-7.97	.301	6.45	.31	-.77	-8.12	.475	3.90
-.83	-1.41	-8.47	.284	6.89	.52	-.90	-8.65	.458	4.06
-.34	-1.70	-8.97	.270	7.34	.75	-1.05	-9.18	.441	4.22
.22	-2.02	-9.47	.257	7.81					

PROPERTIES OF COALESCED JET    X=    .49    Y=    -1.54    Z=    -9.33    U/UJ0=    .54    D= 8.21

XCOORD	YCOORD	ZCOORD	UJ	DIA
.49	-1.54	-9.33	1.000	1.00
.88	-1.77	-9.82	.975	1.10
1.32	-2.02	-10.31	.951	1.23
1.80	-2.29	-10.80	.926	1.40
2.35	-2.57	-11.30	.901	1.62
2.96	-2.89	-11.79	.877	1.70
3.66	-3.23	-12.28	.855	1.76
4.44	-3.61	-12.77	.835	1.83
5.31	-4.02	-13.27	.817	1.90
6.28	-4.46	-13.76	.801	1.97
7.37	-4.95	-14.25	.786	2.04
8.59	-5.48	-14.75	.772	2.11
9.94	-6.06	-15.24	.759	2.18
11.44	-6.70	-15.73	.748	2.25
13.10	-7.40	-16.22	.737	2.32
14.95	-8.16	-16.72	.727	2.39
17.00	-9.00	-17.21	.717	2.47
19.28	-9.91	-17.70	.708	2.54
21.79	-10.92	-18.19	.700	2.62
24.57	-12.02	-18.69	.692	2.69
27.65	-13.23	-19.18	.685	2.77
31.06	-14.56	-19.67	.678	2.85

(c) Jet Centerlines ( Final Iteration)

FIGURE A3. (Continued)

*** INDUCED VELOCITIES AT CONTROL POINTS ***						
PRESSURE COEFFICIENTS AT CONTROL POINTS						
X	Y	Z	CP	U	V	W
-17.500	0.000	0.000	-.32428E-01	.71782E-02	-.37889E-02	.83197E-02
-9.500	0.000	0.000	-.86493E-01	.17141E-01	-.14641E-01	.29449E-01
.500	0.000	0.000	.17890E+00	-.12953E+00	-.12809E+00	-.34906E+00
2.000	0.000	0.000	.47451E-01	-.51691E-01	-.80552E-01	-.23165E+00
-5.500	.500	0.000	-.13475E+00	.24331E-01	-.27676E-01	.10441E+00
-2.500	.500	0.000	.28141E+00	-.12533E+00	-.75878E-01	.99414E-01
1.500	.500	0.000	-.84140E-01	-.45008E-01	-.12990E+00	-.24101E+00
2.000	.500	0.000	-.57098E-01	-.40000E-01	-.11096E+00	-.21444E+00
-8.500	3.000	0.000	-.69511E-01	.17133E-01	-.28554E-02	.32503E-01
-4.500	3.000	0.000	-.31266E+00	.75849E-01	-.32889E-02	.49315E-01
2.500	3.000	0.000	-.31753E+00	.21874E-01	-.12495E+00	-.96571E-01
3.500	3.000	0.000	-.22782E+00	.70633E-02	-.11097E+00	-.92347E-01

(d) Induced Flow Properties (Initial Approximation)

*** INDUCED VELOCITIES AT CONTROL POINTS ***						
PRESSURE COEFFICIENTS AT CONTROL POINTS						
X	Y	Z	CP	U	V	W
-17.500	0.000	0.000	-.39273E-01	.89075E-02	-.39554E-02	.86616E-02
-9.500	0.000	0.000	-.11323E+00	.22725E-01	-.17886E-01	.26050E-01
.500	0.000	0.000	.14731E+00	-.13049E+00	-.14206E+00	-.35394E+00
2.000	0.000	0.000	.10027E-01	-.47171E-01	-.91362E-01	-.20764E+00
-5.500	.500	0.000	-.16747E+00	.27581E-01	-.39784E-01	.94378E-01
-2.500	.500	0.000	.24028E+00	-.13156E+00	-.10375E+00	.78991E-01
1.500	.500	0.000	-.10881E+00	-.41924E-01	-.13572E+00	-.21321E+00
2.000	.500	0.000	-.87509E-01	-.35152E-01	-.11695E+00	-.18980E+00
-8.500	3.000	0.000	-.84400E-01	.19387E-01	-.71775E-02	.30487E-01
-4.500	3.000	0.000	-.30638E+00	.71950E-01	-.10794E-01	.47478E-01
2.500	3.000	0.000	-.31548E+00	.24862E-01	-.11905E+00	-.83084E-01
3.500	3.000	0.000	-.23650E+00	.11939E-01	-.10649E+00	-.81606E-01

(e) Induced Flow Properties (Final Iteration)

FIGURE A3. (Concluded)

## APPENDIX B

### MANUAL FOR COMPUTER PROGRAM

#### Description

The program, which is a modified version of the Jet Flow Field computer program developed by Northrop Corporation under AFFDL contract F33615-69-C-1602 (reference 12), evaluates the induced velocity field due to one or two jets exhausting into an arbitrarily directed crossflow.

The equations of motion governing the development of each jet are integrated numerically for the position of the jet centerline, the nondimensionalized mean jet speed and the nondimensionalized major diameter of the ellipse which represents the jet cross section in the mathematical model. The set of first order differential equations is integrated by means of a fourth order Adams predictor/corrector routine with a Runge-Kutta starting solution.

The induced velocity components due to each jet at a given point are then calculated by replacing each jet with a representative singularity distribution of sinks and doublets along the jet centerline. The contributions to the induced velocity components from the singularity distribution are summed over the length of each jet centerline. The velocity components due to each of the singularity distributions are additive at every control point where induced velocities are to be evaluated.

For the two-jet configuration, the distance between the jet centerlines is tested and when intersection of the two jets is indicated, a coalesced jet is established from continuity and momentum considerations. The coalesced jet is treated as another independent jet in the computations for the induced velocity field.

Jet exit velocity stratification effects may be treated by utilizing the velocity ratio and the jet exit diameter for the equivalent ideal nozzle or, alternatively, by accounting for the internal mixing which takes place through the introduction of input parameters A and B which are described in the discussion of the input data. These approaches to treating different types of jet nozzles producing stratified exit flows are discussed in detail in the section of the report dealing with velocity stratification effects.

For two-jet configurations, an iterative procedure involving modification of the crossflow into which the jets exhaust has been incorporated to account for further mutual interference effects between the two jets. The centerlines of the jets and their representative singularity distributions are calculated, using the unperturbed uniform crossflow in the computations. The jets are then broken into segments and the induced velocity, due to the other jet, is computed for each segment. The crossflow velocity vector for each segment is then modified by this induced velocity and the computations are repeated. In this iteration, and subsequently, each segment is treated as a separate jet exhausting into a uniform crossflow.

### Restrictions

Jets must exhaust at some angle into the crossflow, i. e. the jet exhaust direction may not coincide with the freestream direction.

For a two-jet configuration, the jet exits must both lie in the same XY plane, and the jet exhaust planes, defined by the freestream vector and the initial jet exhaust vectors, must be parallel (see figure B1 for definition of coordinate system).

Control points at which the jet-induced velocity components are to be evaluated may not lie within the jet exhaust itself, as the formulation of the mathematical model is not valid in this region.

### Options

- Induced Velocity Computation: Coordinates of the points at which velocity components are to be evaluated are provided as part of the input to the program. Only the induced velocities are computed at each point specified.
- Induced Pressure Computation: Coordinates of the points at which the induced pressures are to be evaluated are provided as part of the input to the program. In addition to the induced velocity components, the induced pressure in form of the flat plate pressure coefficient is evaluated at each point specified.
- ◆ Note: If it is desired to use this modified version of the Jet Flow Field program in conjunction with the Transformation Method program described in Vol III of reference 12, some minor changes will be required. These consist primarily of including subroutines TRWING, TRBODY and ADAPT as part of the program and providing the input cards of Group B or Group C as described in Section II, Vol. III, reference 12. The program in its present form may be used to generate input to the Lifting Surface program described in Vol. III, reference 12 by exercising the punch control option.

## Operating Information

### Core and Time Requirements:

Computer: CDC 6600  
 Core: 100 K<sub>8</sub> to load  
       61.2 K<sub>8</sub> to execute  
 Time: Approximately one minute for a typical run using 250 control points and two iterations

Additional Requirements: None

### Input Data

The input data cards required by the program are shown in figure B2. The cards of Group I describe the jet configuration and provide parameters needed for computational purposes. The cards of Group II describe the control points at which the jet-induced flow field is to be evaluated.

#### Group I: Description of jet configuration, computational parameters

Card No.	Variable	Format	Description
①	MULT	I6	Specifies number of jets in configuration MULT = 1, 2
	IGEØM	I6	Specifies option of program being exercised If IGEØM $\left\{ \begin{array}{l} = 3 \text{ only induced velocities are evaluated} \\ = 4 \text{ flat plate pressure coefficients are also evaluated} \end{array} \right.$
	IPUNCH	I6	Punch control If IPUNCH $\left\{ \begin{array}{l} = 0 \text{ no punched output} \\ = 1 \text{ punched output generated} \end{array} \right.$
	NSEG	I6	Number of integration intervals per jet segment Limit: $3 \leq \text{NSEG} \leq 10$
	ITER	I6	Number of iterations to be performed on mutual interference velocities. May be left blank for single-jet configurations or two-jet configurations with the jet exits aligned in the freestream direction. For other two-jet configurations, ITER = 2 will normally suffice (see discussion, page 25)

Card No.	Variable	Format	Description
②	ALFA	F12.0	Angle of attack $\alpha$ (see figure B1) }
	BETA	F12.0	Angle of sideslip $\beta$ (see figure B1) } in degrees
③	N	I6	Total number of intervals to be used in the numerical integration of the jet centerline Limit: $N \leq 100$
	G	F12.0	Interval size to be used in the numerical integration of the jet centerline, given as a fraction of the leading jet exit diameter. The integration routine will optimize the actual step size being utilized.
④	XJET	F12.0	X-coordinate of center of jet exit
	YJET	F12.0	Y-coordinate of center of jet exit
	ZJET	F12.0	Z-coordinate of center of jet exit
	PHI	F12.0	Jet exhaust angle $\phi$ (see figure B1) }
	PSI	F12.0	Jet exhaust angle $\psi$ (see figure B1) } in degrees
	DJET	F12.0	Jet exit diameter
⑤	VELJ	F12.0	Freestream to jet exhaust velocity ratio
⑥	A	F12.0	Ratio of effective core diameter to jet exit diameter for annular or vaned nozzles (see discussion, page 11, for details)
	B	F12.0	Jet mixing parameter for annular or vaned nozzles (see page 11)

◆ A and B must be set to zero for a nozzle with uniform exit flow or when treating stratification effects by using an equivalent ideal nozzle. A corresponds to the parameter a, and B corresponds to the parameter b in the discussion of annular and vaned nozzles (pp 10-14).

● Cards 4, 5, 6 are repeated to describe the second jet, if MULT = 2. For two-jet configurations, the upstream jet is listed ahead of the downstream jet.

Card No.	Variable	Format	Description
⑦	DIA	F12.0	Empirical factor for coalesced jet. Function of jet orientation angle $\Omega$ , which is the included angle between line connecting jet exit centers and the freestream velocity vector. If $\Omega < 20^\circ$ , set DIA=1.0. If $\Omega \geq 20^\circ$ , set DIA=0.5. May be left blank for single jet configuration.

Group II: Description of points where induced velocities/and pressures are computed

⑧	NS	I6	Number of spanwise control stations
	NC	I6	Number of control points at each station
⑨	X0(I)	F12.0	X-coordinate of control point
	Y0(I)	F12.0	Y-coordinate of control point
	Z0(I)	F12.0	Z-coordinate of control point

} I = 1, NCxNS  
Limit: I ≤ 600

### Output

Both printed and punched output may be obtained.

#### Printed Output

The jet configuration being treated is identified both by appropriate heading and by printout of pertinent input information. Jet centerline data printed out for all the jets in the configuration, including the jet resulting from the coalescence of two exhausting jets, consists of the centerline coordinates, the nondimensionalized mean jet speed, and the nondimensionalized major diameter of the ellipse representing the jet cross section. The point of intersection of the two exhausting jets in a two-jet configuration is identified, and the initial conditions for the resulting merged jet are given.

The induced velocity components U, V, W, all nondimensionalized by  $U_\infty$ , are printed out for each control point specified as part of the input. Additionally, if IGEOM = 4 was specified, the flat plate pressure coefficient, computed by using an image system, is printed out at each control point.

For a two-jet configuration with the jets not aligned in the freestream direction, where a number of iterations are specified to account for mutual interference effects

between the two jets, the information described above is printed out for each iteration. An example of the printout for a typical problem involving the iterative process to account for mutual interference effects may be found in Appendix A.

### Punched Output

Punched cards may be generated which can be utilized as part of the input to the Lifting Surface program described in reference 12, Vol III. The nondimensionalized velocity component  $W$  is punched out for every control point. This can serve as an approximation to the tangent of the jet-induced downwash angle for small angles of attack. Thus, the punched output from this option can serve as the downwash matrix  $[W]$  in the input to the Lifting Surface program.

## Programming Information

### Logical Structure

The logical flow chart for the program is shown in figure B3.

### Purpose of Subroutines

BITEST	- Tests for blockage and intersection of jets for two-jet configurations
SEGMNT	- Breaks jet into segments for inclusion of mutual interference effects
INTEG	- Integrates equations of motion for the jet path
MØDIFY	- Computes mutually induced velocities
CØMP	- Computes extent of overlap between jets in a two-jet configuration
MUINT	- Computes modified freestream vector
BALANC	- Establishes initial conditions for the coalesced jet from a momentum balance
FIX	- Limits maximum value of mutually induced velocities
ØUTPT ØUTPT1 }	- Transforms local coordinates to program coordinates
VELØC	- Evaluates induced velocities at one control point due to one singularity distribution
DERIV	- Computes derivatives for ADAMS
PRTØUT	- Prints out computed answers
TRANS1 TRANS2 }	- Transforms input coordinates to program coordinates
VEL1	- Computes effective velocity ratio for the downstream jet in a two-jet configuration



TRANS3	-	Transforms program coordinates to output coordinates
PLANE	-	Computes point of intersection between a given plane and a given line
ADAMS	-	Adams predictor/corrector routine
CFCAL CFCAL1	}	Computes direction cosines for jet-centered coordinate system
RØTATE	-	Transforms program coordinates to jet-centered coordinates
XPRØD	-	Computes cross product of two vectors
SØL	-	Solves a system of three simultaneous equations

#### Interdependence of Subroutines

The Calling-Called matrix for the program is shown in figure B4.

[illegible]

FIGURE B2. INPUT DATA



Calling \ Called	MUINT	FIX	BITEST	CFCAL	CFCAL1	INTEG	PRTOUT	ROTATE	TRANS1	TRANS2	TRANS3	SEGMNT	MODIFY	VELOC	VEL1	XPRBD	BALANC	COMP	PLANE	ADAMS	OUTPT	OUTPT1	SOL	DERIV
MAIN	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•									
BITEST				•			•								•	•	•	•						
SEGMNT	•			•	•		•												•	•				
INTEG																			•	•	•			
COMP															•			•						
MODIFY													•											
VELOC							•																	
OUTPT							•																	
OUTPT1							•																	
VEL1															•			•						
PLANE																						•		
ADAMS																							•	
CFCAL															•									
CFCAL1															•									

FIGURE B4. CALLING-CALLED MATRIX

## LISTING OF COMPUTER PROGRAM

1

C  
C  
C  
C  
C

C

1

```

COMMON/BLK14/V2X1,V2Y1,V2Z1,V2X2,V2Y2,V2Z2
COMMON/BLK15/DIR1,DIR2,DIR1T,DIR2T,DR3,ZS01,ZS02,UFACT1,UFACT2
COMMON/BLK16/SDXDZ1,SDXDZ2,SDXDZ3, SXZ1T, SXZ2T
COMMON/BLK17/GS,A,B,C,IS1,IS2,NPS
COMMON/BLK18/U1,V1,W1,U2,V2,W2
COMMON/BLK19/DIARAT,DREF
COMMON/BLK20/A1,B1,A2,B2,ZSTOR1,ZSTOR2

```

C

```

DIMENSION XO(600),YO(600),ZO(600),U(600),V(600),W(600)
DIMENSION CP(600)
DIMENSION PHID(3),PSID(3)

```

C

C

C

SET PARAMETERS

```

E1 = .45
E2 = .08
E3 = 30.
PI = 3.1416
C1 = 2.24

```

C

C

C

READ IN JET DATA

```

READ (5,501) MULT,IGEOM,IPUNCH,NPS,NOIT
READ (5,502) ALFA,BETA
READ (5,503) N,GS

```

501 FORMAT (12I6)

502 FORMAT (6F12.0)

503 FORMAT (16,F12.0)

```

READ (5,502) XJ1(1),YJ1(1),ZJ1(1),PHID(1),PSID(1),DJET1(1),
1 VELJ1(1)

```

```

READ (5,502) A1,B1

```

```

IF (MULT-2) 4,2,2

```

```

2 READ (5,502) XJ2(1),YJ2(1),ZJ2(1),PHID(2),PSID(2),DJET2(1),
1 VELJ2(1)

```

```

READ (5,502) A2,B2

```

4 CONTINUE

```

READ (5,502) DIARAT

```

```

WRITE (6,690)

```

690 FORMAT (1H1)

```

IF (MULT-2) 14,15,15

```

14 WRITE (6,603)

603 FORMAT (1H0,44X,32H\*\*\* SINGLE JET CONFIGURATION \*\*\*/)

```

N1 = N+1

```

```

GO TO 17

```

15 WRITE (6,604)

604 FORMAT (1H0,45X,29H\*\*\* TWO-JET CONFIGURATION \*\*\*/)

17 CONTINUE

```

WRITE (6,606) XJ1(1),YJ1(1),ZJ1(1),PHID(1),PSID(1),VELJ1(1),
1 DJET1(1)

```

606 FORMAT (1H0,15X,4HXJET,11X,4HYJET,11X,4HZJET,12X,3HPhi,12X,3HPSI,  
112X,5HU/UJ0,11X,2HDO/8X,F15.4,1X,F14.4,1X,F14.4,1X,F14.4,1X,F14.4,  
21X,F14.4,1X,F14.4)

```

IF (MULT-2) 20,18,18

```

```

18 WRITE (6,607) XJ2(1),YJ2(1),ZJ2(1),PHID(2),PSID(2),VELJ2(1),
1 DJET2(1)

```

```

607  FORMAT (8X,F15.4,1X,F14.4,1X,F14.4,1X,F14.4,1X,F14.4,1X,F14.4,1X,
      1 F14.4)
20   CONTINUE
      IF (A1) 41,41,42
41   IF (MULT-2) 43,44,44
43   WRITE (6,610)
      GO TO 56
610  FORMAT (1H0,/15X,44HNOZZLE HAS UNIFORM EXIT FLOW CHARACTERISTICS)
44   IF (A2) 45,45,46
45   WRITE (6,611)
      GO TO 56
611  FORMAT (1H0,/15X,51HBOTH NOZZLES HAVE UNIFORM EXIT FLOW CHARACTERI
      1STICS)
46   WRITE (6,612) A2,B2
      GO TO 56
612  FORMAT (1H0,/15X,53HNOZZLE OF JET 1 HAS UNIFORM EXIT FLOW CHARACTE
      1RISTICS/15X,52HNONUNIFORM EXIT FLOW PARAMETERS FOR NOZZLE OF JET 2
      2:,3X,4HA2 =,F5.3,3X,4HB2 =,F6.3)
42   IF (MULT-2) 47,48,48
47   WRITE (6,613) A1,B1
      GO TO 56
613  FORMAT (1H0,/15X,47HNONUNIFORM EXIT FLOW PARAMETERS FOR THE NOZZLE
      1:,3X,4HA1 =,F5.3,3X,4HB1 =,F6.3)
48   IF (A2) 49,49,55
49   WRITE (6,614) A1,B1
      GO TO 56
614  FORMAT (1H0,/15X,52HNONUNIFORM EXIT FLOW PARAMETERS FOR NOZZLE OF
      1JET 1:,3X,4HA1 =,F5.3,3X,4HB1 =,F6.3/15X,53HNOZZLE OF JET 2 HAS UN
      2IFORM EXIT FLOW CHARACTERISTICS)
55   WRITE (6,615) A1,B1,A2,B2
615  FORMAT (1H0,/15X,52HNONUNIFORM EXIT FLOW PARAMETERS FOR NOZZLE OF
      1JET 1:,3X,4HA1 =,F5.3,3X,4HB1 =,F6.3/15X,52HNONUNIFORM EXIT FLOW P
      2ARAMETERS FOR NOZZLE OF JET 2:,3X,4HA2 =,F5.3,3X,4HB2 =,F6.3)
56   CONTINUE
      WRITE (6,608) ALFA,BETA
608  FORMAT(1H0,/15X,19HANGLE OF ATTACK   =,1X,F7.2/15X,19HANGLE OF SID
      1ESLIP =,1X,F7.2)
      WRITE (6,609) N,GS
609  FORMAT(1H0,/15X,32HNUMBER OF STEPS IN INTEGRATION =,1X,I3,/15X,22H
      1INTEGRATION INTERVAL =,1X,F5.2,1X,18HJET EXIT DIAMETERS)
      IF (MULT-2) 58,57,57
57   WRITE (6,616) DIARAT
616  FORMAT (1H0,/15X,88HINITIAL RATIO OF MINOR TO MAJOR DIAMETER OF EL
      1LIPTICAL CROSS SECTION OF COALESCED JET IS,F4.1)
58   CONTINUE
      CALL TRANS1 (MULT,ALFA,BETA,PSID)
      DO 31 I=1,10
          U1(I) = 0.
          V1(I) = 0.
          W1(I) = 0.
          U2(I) = 0.
          V2(I) = 0.
31   W2(I) = 0.
      ITER = 0

```

```

C
C   ***  START OF ITERATION LOOP  ***
C
30  CONTINUE
    DO 8 I=1,MULT
      PHI = PHID(I)*.0174533
      PSI = PSID(I)*.0174533
      IF (I-2) 5,6,6
5    CONTINUE
      V2X1 = SIN(PHI)*COS(PSI)
      V2Y1 = COS(PHI)
      V2Z1 = SIN(PHI)*SIN(PSI)
      CALL MUINT (ALFQ,BETQ,GETQ,U1(1),V1(1),W1(1),ALFQ1(1),BETQ1(1),
1 GETQ1(1),UFACT1(1))
      CALL CFCAL1 (ALFQ1(1),BETQ1(1),GETQ1(1),V2X1,V2Y1,V2Z1,CF1,1)
      CALL ROTATE (V2X1,V2Y1,V2Z1,CF1(1,1,1),VXT,VYT,VZT,0)
      UJ1(1,1) = 1.
      D1(1,1) = 1.
      X1(1,1) = 0.
      Z1(1,1) = 0.
      DXDZ1(1,1) = VXT/VZT
      XBAS1(1) = XJ1(1)
      YBAS1(1) = YJ1(1)
      ZBAS1(1) = ZJ1(1)
      IF (ITER) 34,33,34
33  A = CF1(3,1,1)
      B = CF1(3,2,1)
      C = CF1(3,3,1)
34  CONTINUE
      COSTHP = 1./SQRT(1.+(VXT/VZT)**2)
      COSTH = A*V2X1 + B*V2Y1 + C*V2Z1
      G = GS
      G = G*COSTHP/COSTH
      STEPI = .2*G
      DIR1(1) = 1.
      ZS01(1) = 0.
      B1 = B1/COSTHP
      D = ATAN(VXT/VZT)
      IF (VXT) 901,902,902
901 F1 = .3*COS(D)
      GO TO 903
902 F1 = .3/COS(D)
903 CONTINUE
      GO TO 8
6    CONTINUE
      V2X2 = SIN(PHI)*COS(PSI)
      V2Y2 = COS(PHI)
      V2Z2 = SIN(PHI)*SIN(PSI)
      CALL MUINT (ALFQ,BETQ,GETQ,U2(1),V2(1),W2(1),ALFQ2(1),BETQ2(1),
1 GETQ2(1),UFACT2(1))
      CALL CFCAL1 (ALFQ2(1),BETQ2(1),GETQ2(1),V2X2,V2Y2,V2Z2,CF2,1)
      CALL ROTATE (V2X2,V2Y2,V2Z2,CF2(1,1,1),VXT,VYT,VZT,0)
      UJ2(1,1) = 1.
      D2(1,1) = 1.

```



```

X2(1,1) = 0.
Z2(1,1) = 0.
DXDZ2(1,1) = VXT/VZT
XBAS2(1) = XJ2(1)
YBAS2(1) = YJ2(1)
ZBAS2(1) = ZJ2(1)
COSTHP = 1./SQRT(1.+(VXT/VZT)**2)
COSTH = A*V2X2 +B*V2Y2 +C*V2Z2
G2 = GS*DJET1(1)/DJET2(1)
G2 = G2*COSTHP/COSTH
STEPI2 = .2*G2
DIR2(1) = 1.
ZS02(1) = 0.
B2 = B2/COSTHP
D = ATAN(VXT/VZT)
IF (VXT) 904,905,905
904 F2 = .3*COS(D)
GO TO 906
905 F2 = .3/COS(D)
906 CONTINUE
8 CONTINUE
IF (ITER) 9,7,9
7 CALL VEL1 (MULT,ALFA,VK1)
IF (MULT-2) 12,11,11
11 CONTINUE
C
C
C COMPUTE INITIAL OVERLAP
CALL XPROD (V2X1,V2Y1,V2Z1,ALFQ,BETQ,GETQ,CFNX,CFNY,CFNZ)
CALL XPROD (V2X2,V2Y2,V2Z2,ALFQ,BETQ,GETQ,XT2,YT2,ZT2)
CALL PLANE (CFNX,CFNY,CFNZ,XJ1(1),YJ1(1),ZJ1(1),XT2,YT2,ZT2,
1 XJ2(1),YJ2(1),ZJ2(1),XI,YI,ZI)
DIST = SQRT((XI-XJ2(1))**2 +(YI-YJ2(1))**2 +(ZI-ZJ2(1))**2)
R = DJET1(1)*.5-DIST
FACT = (1.0+R/(DJET2(1)*.5))* .5
IF (FACT-1.) 3,10,10
3 IF (FACT) 10,10,13
13 TEST1 = DJET1(1)*.5+DIST
TEST2 = DJET2(1)*.5
IF (TEST1-TEST2) 23,10,10
23 FACT = DJET1(1)/DJET2(1)
10 OVLP = 1.-FACT
GO TO 9
12 OVLP = 0.
9 CONTINUE
PAR(1) = E1
PAR(2) = E2
PAR(3) = E3
PAR(7) = PI
PAR(8) = C1
PAR(9) = 1.
PAR(11) = A1
PAR(12) = B1
PAR(13) = A2

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```

PAR(14) = B2
ZSTOR1 = 0.
ZSTOR2 = 0.
N2 = 0
N3 = 0
IHOLD1 = 0
KOUNT1 = 0
TNEG = BETQ*V2Y1
DREF = DJET1(1)
IFIX1 = 0
KSEG = 1
DO 50 I=1,N
  IONE = I
  ITWO = I
  VKONST = VK1
  IF (MULT-2) 24,22,22
24  IS1 = I-(KSEG-1)*NPS
  GO TO 27
22  IF (IHOLD1-1) 25,25,21
C
C    TESTS FOR BLOCKAGE AND INTERSECTION,PART OF INTEGRATION LOOP
C
25  CALL BITEST (I,TNEG,KSEG)
  IF (IFIX1) 27,27,26
26  CALL SEGMNT (I,KSEG,1)
  NL = IFIX1-(KSEG-1)*NPS
21  CALL SEGMNT (I,KSEG,2)
27  CONTINUE
C
C    INTEGRATION OF THE EQUATIONS OF MOTION FOR THE JET PATH
C
  CALL INTEG (I,TNEG,KSEG)
  IF (IHOLD1-2) 28,50,50
28  IF (I-N) 29,50,50
29  IF (I-KSEG*NPS) 50,40,40
40  CALL SEGMNT (I,KSEG,3)
50  CONTINUE
  IF (IFIX1) 51,51,52
51  NL = IS1+1
52  IF (MULT-2) 60,53,53
53  IF (OVLP-.01) 60,60,54
54  CONTINUE
  CALL MODIFY (KSEG,TNEG,NL)
  CALL FIX (U1,V1,W1,U2,V2,W2,KSEG)
  IF (ITER) 60,60,70
60  CONTINUE
C
C    READING IN CONTROL POINTS WHERE INDUCED VELOCITIES WILL BE COMPUTE
C
  READ (5,501) NSMAX,NC
  NK = NSMAX*NC
  READ (5,502) (X0(I),Y0(I),Z0(I), I=1,NK)
70  CONTINUE
  CALL TRANS2 (Y0,Z0,NK)

```

```

C
C      EVALUATE INDUCED VELOCITIES AT EACH CONTROL POINT
C
      IF (MULT-2) 90,91,91
91    KTR1 = 0
      KTR2 = 0
      IF (TNEG) 96,96,97
96    KTR2 = KOUNT1
      GO TO 90
97    KTR1 = KOUNT1
90    CONTINUE
      IF (NL-2) 71,71,72
71    KSEG1 = KSEG-1
      GO TO 75
72    KSEG1 = KSEG
75    CONTINUE
      DO 80 J=1,NK
      U(J) = 0.
      V(J) = 0.
      W(J) = 0.
      DO 80 I=1,KSEG1
      PAR(6) = VELJ1(I)
      PAR(5) = F1
      PAR(9) = DIR1(I)
      IF (MULT-2) 81,82,82
81    IF (I-KSEG) 113,114,114
82    IF (I-1) 111,112,111
112   NF = NPS+1-KTR1
      GO TO 83
111   IF (I-KSEG) 113,114,114
113   NF = NPS+1
      GO TO 83
114   NF = NL
83    CONTINUE
      CALL VELOC (1,NF,Z1(1,I),X1(1,I),DXDZ1(1,I),UJ1(1,I),D1(1,I),
1     UUE1(1,I),XJ1(I),YJ1(I),ZJ1(I),DJET1(I),CF1(1,1,I),PAR,XO(J),
2     YO(J),ZO(J),UIND,VIND,WIND,SDXDZ1(1,I))
      U(J) = U(J)+UIND
      V(J) = V(J)+VIND
      W(J) = W(J)+WIND
      IF (MULT-2) 80,86,86
86    CONTINUE
      PAR(6) = VELJ2(I)
      PAR(5) = F2
      PAR(9) = DIR2(I)
      IF (I-1) 87,115,87
115   NF = NPS+1-KTR2
87    CONTINUE
      CALL VELOC (1,NF,Z2(1,I),X2(1,I),DXDZ2(1,I),UJ2(1,I),D2(1,I),
1     UUE2(1,I),XJ2(I),YJ2(I),ZJ2(I),DJET2(I),CF2(1,1,I),PAR,XO(J),
2     YO(J),ZO(J),UIND,VIND,WIND,SDXDZ2(1,I))
      U(J) = U(J)+UIND
      V(J) = V(J)+VIND
      W(J) = W(J)+WIND
80    CONTINUE

```

```

      IF (IHOLD1-1) 88,88,89
89  CONTINUE
      PAR(6) = VELJ3
      PAR(5) = F3
      PAR(9) = DR3
      N3 = ITHR+1
      DO 120 J=1,NK
      CALL VELOC (1,N3,Z3,X3,DXDZ3,UJ3,D3,UUE3,XJ3,YJ3,ZJ3,DJET3,
1  CF3,PAR,XO(J),YO(J),ZO(J),UIND,VIND,WIND,SDXDZ3)
      U(J) = U(J)+UIND
      V(J) = V(J)+VIND
120  W(J) = W(J)+WIND
88  CONTINUE
      IF (IGEOM-3) 93,93,92
C
C  COMPUTE FLAT PLATE PRESSURE COEFFICIENTS
C
92  DO 85 J=1,NK
      CPT = 4.*(U(J)*(ALFQ+U(J))+W(J)*(GETQ+W(J)))
85  CP(J) = 1.-(ALFQ*ALFQ +GETQ*GETQ +CPT)
93  CONTINUE
      CALL TRANS3 (YO,ZO,V,W,NK,KSEG,NPS,TNEG,NL)
C
C  PRINT OUT COMPUTED RESULTS
C
      CALL PRTOU (IGEOM,XO,YO,ZO,U,V,W,CP,NK,ITER,NOIT,OVLP)
      IF (OVLP-.01) 94,94,98
98  IF (OVLP-.99) 77,77,79
77  DO 78 I=1,KSEG
      U1(I) = U1(I)*OVLP
      V1(I) = V1(I)*OVLP
      W1(I) = W1(I)*OVLP
      U2(I) = U2(I)*OVLP
      V2(I) = V2(I)*OVLP
78  W2(I) = W2(I)*OVLP
79  CONTINUE
      IF (ITER-NOIT) 35,94,94
35  ITER = ITER+1
      GO TO 30
C
C  ***  END OF ITERATION LOOP  ***
C
94  CONTINUE
      IF (IPUNCH) 95,99,95
C
C  PUNCH OUT DATA FOR LIFTING SURFACE PROGRAM
C
95  DO 101 I=1,NK
101  W(I) = -W(I)
      J1 = 1

```

```

DO 102 I=1,NSMAX
J2 = J1+NC-1
WRITE (7,710) (W(J),J=J1,J2)
102 J1 = J2+1
710 FORMAT (5E14.7)
99 CONTINUE
STOP
END

```

# SUBROUTINE SEGMENT (I,KSEG,IND)

```

C
C IND=1, ESTABLISHES INITIAL CONDITIONS FOR CONTINUATION JETS
C IND=2, INTEGRATES CONTINUATION JETS
C IND=3, ESTABLISHES INITIAL CONDITIONS FOR A NEW SEGMENT
C

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## EXTERNAL DERIV

```

C
DIMENSION X1(11,10),Z1(11,10),UJ1(11,10),D1(11,10),DXDZ1(11,10)
DIMENSION X2(11,10),Z2(11,10),UJ2(11,10),D2(11,10),DXDZ2(11,10)
DIMENSION X1T(100),Z1T(100),UJ1T(100),D1T(100),DXDZ1T(100)
DIMENSION X2T(100),Z2T(100),UJ2T(100),D2T(100),DXDZ2T(100)
DIMENSION X3(100),Z3(100),UJ3(100),D3(100),DXDZ3(100)
DIMENSION XBAS1(100),YBAS1(100),ZBAS1(100)
DIMENSION XBAS2(100),YBAS2(100),ZBAS2(100)
DIMENSION XBS1T(100),YBS1T(100),ZBS1T(100)
DIMENSION XBS2T(100),YBS2T(100),ZBS2T(100)
DIMENSION XBAS3(100),YBAS3(100),ZBAS3(100)
DIMENSION CF1(3,3,10),CF2(3,3,10),CF1T(3,3),CF2T(3,3),CF3(3,3)
DIMENSION UUE1(11,10),UUE2(11,10),UUE1T(100),UUE2T(100),UUE3(100)
DIMENSION SDXDZ1(11,10),SDXDZ2(11,10),SXZ1T(100),SXZ2T(100),
1 SDXDZ3(100)
DIMENSION PAR(15)
DIMENSION XJ1(10),YJ1(10),ZJ1(10),DJET1(10),VELJ1(10)
DIMENSION XJ2(10),YJ2(10),ZJ2(10),DJET2(10),VELJ2(10)
DIMENSION ALFQ1(10),BETQ1(10),GETQ1(10),ALFQ2(10),BETQ2(10),
1 GETQ2(10)
DIMENSION DIR1(10),DIR2(10),ZS01(10),ZS02(10),UFACT1(10),
1 UFACT2(10)
DIMENSION U1(10),V1(10),W1(10),U2(10),V2(10),W2(10)
C
COMMON/BLK1/CF1,CF2,CF1T,CF2T,CF3,UUE1,UUE2,UUE1T,UUE2T,UUE3,PAR
COMMON/BLK2/X1,Z1,UJ1,D1,DXDZ1,X2,Z2,UJ2,D2,DXDZ2
COMMON/BLK3/X1T,Z1T,UJ1T,D1T,DXDZ1T,X2T,Z2T,UJ2T,D2T,DXDZ2T
COMMON/BLK4/X3,Z3,UJ3,D3,DXDZ3
COMMON/BLK5/XBAS1,YBAS1,ZBAS1,XBAS2,YBAS2,ZBAS2,XBAS3,YBAS3,ZBAS3
COMMON/BLK6/XBS1T,YBS1T,ZBS1T,XBS2T,YBS2T,ZBS2T
COMMON/BLK7/ALFQ,BETQ,GETQ,F1,F2,F3,VKONST
COMMON/BLK8/ALFQ1,BETQ1,GETQ1,ALFQ2,BETQ2,GETQ2
COMMON/BLK9/MULT,IHOLD1,KOUNT1,IONE,ITWO,ITHR,N1,N2,N3,IFIX1
COMMON/BLK10/XJ1,YJ1,ZJ1,DJET1,VELJ1,XJ2,YJ2,ZJ2,DJET2,VELJ2
COMMON/BLK11/XJ1T,YJ1T,ZJ1T,DJET1T,VELJ1T,XJ2T,YJ2T,ZJ2T,DJET2T,
1 VELJ2T
COMMON/BLK12/XJ3,YJ3,ZJ3,DJET3,VELJ3

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```

COMMON/BLK13/G,G2,G3,STEP1,STEP12,STEP13
COMMON/BLK14/V2X1,V2Y1,V2Z1,V2X2,V2Y2,V2Z2
COMMON/BLK15/DIR1,DIR2,DIR1T,DIR2T,DR3,ZS01,ZS02,UFACT1,UFACT2
COMMON/BLK16/SDXDZ1,SDXDZ2,SDXDZ3,SXZ1T,SXZ2T
COMMON/BLK17/GS,A,B,C,IS1,IS2,NPS
COMMON/BLK18/U1,V1,W1,U2,V2,W2
COMMON/BLK20/A1,B1,A2,B2,ZSTOR1,ZSTOR2

```

C

```

DIMENSION FIN(4),FOUT(4)

```

C

```

IF (IND-2) 26,21,40
26 NL = IFIX1-(KSEG-1)*NPS
   XJ1T = XBAS1(IONE)
   YJ1T = YBAS1(IONE)
   ZJ1T = ZBAS1(IONE)
   UJ1T(1) = 1.
   D1T(1) = 1.
   X1T(1) = 0.
   Z1T(1) = 0.
   CALL CFCAL (ALFQ,BETQ,GETQ,V2X1,V2Y1,V2Z1,CF1T)
   CALL ROTATE (V2X1,V2Y1,V2Z1,CF1T,VXT,VYT,VZT,0)
   DXDZ1T(1) = VXT/VZT
   XBS1T(1) = XJ1T
   YBS1T(1) = YJ1T
   ZBS1T(1) = ZJ1T
   DJET1T = D1(NL,KSEG)*DJET1(KSEG)
   VELJ1T = UJ1(NL,KSEG)*VELJ1(KSEG)
   G1T = GS*DJET1(1)/DJET1T
   STEP1T = G1T*.2
   IF (DIR1(KSEG)-.2501) 320,320,321
321 ZOVM = (ZS01(KSEG)+Z1(NL,KSEG))/(VELJ1(KSEG)*UUE1(NL-1,KSEG))
   IF (ZOVM-F1) 314,314,320
314 DIR1T = 1.-.75*ZOVM/F1
   GO TO 325
320 DIR1T = .25
325 CONTINUE
   XJ2T = XBAS2(ITWO)
   YJ2T = YBAS2(ITWO)
   ZJ2T = ZBAS2(ITWO)
   UJ2T(1) = 1.
   D2T(1) = 1.
   X2T(1) = 0.
   Z2T(1) = 0.
   CALL CFCAL (ALFQ,BETQ,GETQ,V2X2,V2Y2,V2Z2,CF2T)
   CALL ROTATE(V2X2,V2Y2,V2Z2,CF2T,VXT,VYT,VZT,0)
   DXDZ2T(1) = VXT/VZT
   XBS2T(1) = XJ2T
   YBS2T(1) = YJ2T
   ZBS2T(1) = ZJ2T
   DJET2T = D2(NL,KSEG)*DJET2(KSEG)
   VELJ2T = UJ2(NL,KSEG)*VELJ2(KSEG)
   G2T = GS*DJET1(1)/DJET2T
   STEP2T = G2T*.2

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```

      IF (DIR2(KSEG)-.2501) 330,330,331
331  ZOVM = (ZS02(KSEG)+Z2(NL,KSEG))/(VELJ2(KSEG)*UUE2(NL-1,KSEG))
      IF (ZOVM-F2) 324,324,330
324  DIR2T = 1.-.75*ZOVM/F2
      GO TO 335
330  DIR2T = .25
335  CONTINUE
      GO TO 50
21  CONTINUE
      IA = I-IFIX1+1
      PAR(6) = VELJ1T
      PAR(5) = F1
      PAR(9) = DIR1T
      PAR(15) = 3.
      UUE1T(IA) = 1.
      Z1T(IA+1) = Z1T(IA)+G1T
      FIN(1) = UJ1T(IA)
      FIN(2) = D1T(IA)
      FIN(3) = X1T(IA)
      FIN(4) = DXDZ1T(IA)
      CALL ADAMS (4,Z1T(IA),Z1T(IA+1),STEP1T,G1T,999,1.0E-04,1.0E-05,0,
1  FIN,FOUT,PAR,DERIV)
      UJ1T(IA+1) = FOUT(1)
      D1T(IA+1) = FOUT(2)
      X1T(IA+1) = FOUT(3)
      DXDZ1T(IA+1) = FOUT(4)
      SXZ1T(IA+1) = PAR(10)
      CALL OUTPT (X1T(IA+1),Z1T(IA+1),DXDZ1T(IA+1),CF1T,DJET1T,XJ1T,
1  YJ1T,ZJ1T,XBS1T(IA+1),YBS1T(IA+1),ZBS1T(IA+1),DUM,DUM,DUM)
      PAR(6) = VELJ2T
      PAR(5) = F2
      PAR(9) = DIR2T
      PAR(15) = 3.
      UUE2T(IA) = 1.
      Z2T(IA+1) = Z2T(IA)+G2T
      FIN(1) = UJ2T(IA)
      FIN(2) = D2T(IA)
      FIN(3) = X2T(IA)
      FIN(4) = DXDZ2T(IA)
      CALL ADAMS (4,Z2T(IA),Z2T(IA+1),STEP2T,G2T,999,1.0E-04,1.0E-05,0,
1  FIN,FOUT,PAR,DERIV)
      UJ2T(IA+1) = FOUT(1)
      D2T(IA+1) = FOUT(2)
      X2T(IA+1) = FOUT(3)
      DXDZ2T(IA+1) = FOUT(4)
      SXZ2T(IA+1) = PAR(10)
      CALL OUTPT (X2T(IA+1),Z2T(IA+1),DXDZ2T(IA+1),CF2T,DJET2T,XJ2T,
1  YJ2T,ZJ2T,XBS2T(IA+1),YBS2T(IA+1),ZBS2T(IA+1),DUM,DUM,DUM)
      GO TO 50
40  CONTINUE
      KSEG = KSEG+1
      XJ1(KSEG) = XBAS1(IONE+1)
      YJ1(KSEG) = YBAS1(IONE+1)
      ZJ1(KSEG) = ZBAS1(IONE+1)

```

```

UJ1(1,KSEG) = 1.
D1(1,KSEG) = 1.
X1(1,KSEG) = 0.
Z1(1,KSEG) = 0.
CALL MUINT (ALFQ,BETQ,GETQ,U1(KSEG),V1(KSEG),W1(KSEG),ALFQ1(KSEG),
1 BETQ1(KSEG),GETQ1(KSEG),UFACT1(KSEG))
CALL CFCAL1 (ALFQ1(KSEG),BETQ1(KSEG),GETQ1(KSEG),V2X1,V2Y1,V2Z1,
1 CF1,KSEG)
CALL ROTATE (V2X1,V2Y1,V2Z1,CF1(1,1,KSEG),VXT,VYT,VZT,0)
DXDZ1(1,KSEG) = VXT/VZT
DJET1(KSEG) = D1(IS1+1,KSEG-1)*DJET1(KSEG-1)
VELJ1(KSEG) = UJ1(IS1+1,KSEG-1)*VELJ1(KSEG-1)
COSTHP = 1./SQRT(1.+(VXT/VZT)**2)
COSTH = A*V2X1 +B*V2Y1 +C*V2Z1
G = GS*DJET1(1)/DJET1(KSEG)
G = G*COSTHP/COSTH
STEP1 = G*.2
IF (PAR(11)) 423,423,422
422 ZSTOR1 = ZSTOR1+Z1(IS1+1,KSEG-1)*DJET1(KSEG-1)/DJET1(1)
PAR(11) = (A1-B1*ZSTOR1)*DJET1(1)/DJET1(KSEG)
423 CONTINUE
IF (DIR1(KSEG-1)-.2501) 420,420,421
421 ZOVM = (ZS01(KSEG-1)+Z1(IS1+1,KSEG-1))/(VELJ1(KSEG-1)*
1 UUE1(IS1,KSEG-1))
IF (ZOVM-F1) 414,414,420
414 DIR1(KSEG) = 1.-.75*ZOVM/F1
ZS01(KSEG) = (1.-DIR1(KSEG))*VELJ1(KSEG)*UFACT1(KSEG)*
1 UUE1(IS1,KSEG-1)/UFACT1(KSEG-1)*F1/.75
GO TO 425
420 DIR1(KSEG) = .25
425 CONTINUE
IF (MULT-2) 50,450,450
450 CONTINUE
XJ2(KSEG) = XBAS2(ITWO+1)
YJ2(KSEG) = YBAS2(ITWO+1)
ZJ2(KSEG) = ZBAS2(ITWO+1)
UJ2(1,KSEG) = 1.
D2(1,KSEG) = 1.
X2(1,KSEG) = 0.
Z2(1,KSEG) = 0.
CALL MUINT (ALFQ,BETQ,GETQ,U2(KSEG),V2(KSEG),W2(KSEG),ALFQ2(KSEG),
1 BETQ2(KSEG),GETQ2(KSEG),UFACT2(KSEG))
CALL CFCAL1 (ALFQ2(KSEG),BETQ2(KSEG),GETQ2(KSEG),V2X2,V2Y2,V2Z2,
1 CF2,KSEG)
CALL ROTATE (V2X2,V2Y2,V2Z2,CF2(1,1,KSEG),VXT,VYT,VZT,0)
DXDZ2(1,KSEG) = VXT/VZT
DJET2(KSEG) = D2(IS2+1,KSEG-1)*DJET2(KSEG-1)
VELJ2(KSEG) = UJ2(IS2+1,KSEG-1)*VELJ2(KSEG-1)
COSTHP = 1./SQRT(1.+(VXT/VZT)**2)
COSTH = A*V2X2 +B*V2Y2 +C*V2Z2
G2 = GS*DJET1(1)/DJET2(KSEG)
G2 = G2*COSTHP/COSTH
STEP12 = G2*.2

```



```

      IF (PAR(13)) 433,433,432
432  ZSTOR2 = ZSTOR2+Z2(IS2+1,KSEG-1)*DJET2(KSEG-1)/DJET2(1)
      PAR(13) = (A2-B2*ZSTOR2)*DJET2(1)/DJET2(KSEG)
433  CONTINUE
      IF (DIR2(KSEG-1)-.2501) 430,430,431
431  ZOVM = (ZS02(KSEG-1)+Z2(IS2+1,KSEG-1))/(VELJ2(KSEG-1)*
1      UUE2(IS2,KSEG-1))
      IF (ZOVM-F2) 424,424,430
424  DIR2(KSEG) = 1.-.75*ZOVM/F2
      ZS02(KSEG) = (1.-DIR2(KSEG))*VELJ2(KSEG)*UFACT2(KSEG)*
1      UUE2(IS2,KSEG-1)/UFACT2(KSEG-1)*F2/.75
      GO TO 435
430  DIR2(KSEG) = .25
435  CONTINUE
50   CONTINUE
      RETURN
      END

```

#### SUBROUTINE MODIFY (KSEG,TNEG,NL)

COMPUTES MUTUALLLY INDUCED VELOCITIES

```

C
C
C
DIMENSION X1(11,10),Z1(11,10),UJ1(11,10),D1(11,10),DXDZ1(11,10)
DIMENSION X2(11,10),Z2(11,10),UJ2(11,10),D2(11,10),DXDZ2(11,10)
DIMENSION X1T(100),Z1T(100),UJ1T(100),D1T(100),DXDZ1T(100)
DIMENSION X2T(100),Z2T(100),UJ2T(100),D2T(100),DXDZ2T(100)
DIMENSION X3(100),Z3(100),UJ3(100),D3(100),DXDZ3(100)
DIMENSION XBAS1(100),YBAS1(100),ZBAS1(100)
DIMENSION XBAS2(100),YBAS2(100),ZBAS2(100)
DIMENSION XBS1T(100),YBS1T(100),ZBS1T(100)
DIMENSION XBS2T(100),YBS2T(100),ZBS2T(100)
DIMENSION XBAS3(100),YBAS3(100),ZBAS3(100)
DIMENSION CF1(3,3,10),CF2(3,3,10),CF1T(3,3),CF2T(3,3),CF3(3,3)
DIMENSION UUE1(11,10),UUE2(11,10),UUE1T(100),UUE2T(100),UUE3(100)
DIMENSION SDXDZ1(11,10),SDXDZ2(11,10),SXZ1T(100),SXZ2T(100),
1 SDXDZ3(100)
DIMENSION PAR(15)
DIMENSION XJ1(10),YJ1(10),ZJ1(10),DJET1(10),VELJ1(10)
DIMENSION XJ2(10),YJ2(10),ZJ2(10),DJET2(10),VELJ2(10)
DIMENSION ALFQ1(10),BETQ1(10),GETQ1(10),ALFQ2(10),BETQ2(10),
1 GETQ2(10)
DIMENSION DIR1(10),DIR2(10),ZS01(10),ZS02(10),UFACT1(10),
1 UFACT2(10)
DIMENSION U1(10),V1(10),W1(10),U2(10),V2(10),W2(10)
C
COMMON/BLK1/CF1,CF2,CF1T,CF2T,CF3,UUE1,UUE2,UUE1T,UUE2T,UUE3,PAR
COMMON/BLK2/X1,Z1,UJ1,D1,DXDZ1,X2,Z2,UJ2,D2,DXDZ2
COMMON/BLK3/X1T,Z1T,UJ1T,D1T,DXDZ1T,X2T,Z2T,UJ2T,D2T,DXDZ2T
COMMON/BLK4/X3,Z3,UJ3,D3,DXDZ3
COMMON/BLK5/XBAS1,YBAS1,ZBAS1,XBAS2,YBAS2,ZBAS2,XBAS3,YBAS3,ZBAS3
COMMON/BLK6/XBS1T,YBS1T,ZBS1T,XBS2T,YBS2T,ZBS2T
COMMON/BLK7/ALFQ,BETQ,GETQ,F1,F2,F3,VKONST
COMMON/BLK8/ALFQ1,BETQ1,GETQ1,ALFQ2,BETQ2,GETQ2

```

```

COMMON/BLK9/MULT,IHOLD1,KOUNT1,IONE,ITWO,ITHR,N1,N2,N3,IFIX1
COMMON/BLK10/XJ1,YJ1,ZJ1,DJET1,VELJ1,XJ2,YJ2,ZJ2,DJET2,VELJ2
COMMON/BLK11/XJ1T,YJ1T,ZJ1T,DJET1T,VELJ1T,XJ2T,YJ2T,ZJ2T,DJET2T,
1      VELJ2T
COMMON/BLK12/XJ3,YJ3,ZJ3,DJET3,VELJ3
COMMON/BLK13/G,G2,G3,STEP1,STEP12,STEP13
COMMON/BLK14/V2X1,V2Y1,V2Z1,V2X2,V2Y2,V2Z2
COMMON/BLK15/DIR1,DIR2,DIR1T,DIR2T,DR3,ZS01,ZS02,UFACT1,UFACT2
COMMON/BLK16/SDXDZ1,SDXDZ2,SDXDZ3,SXZ1T,SXZ2T
COMMON/BLK17/GS,A,B,C,IS1,IS2,NPS
COMMON/BLK18/U1,V1,W1,U2,V2,W2

```

```

C      DIMENSION XP1(10),YP1(10),ZP1(10),XP2(10),YP2(10),ZP2(10)

```

```

C      CHOOSING POINTS ON CENTERLINE

```

```

C      KTR1 = 0
C      KTR2 = 0
C      IF (TNEG) 206,206,207
206    KTR2 = KOUNT1
      GO TO 208
207    KTR1 = KOUNT1
208    CONTINUE
      DO 70 I=1,KSEG
        IF (I-1) 71,71,72
71      IK = 1
        GO TO 73
72      IK = (I-1)*NPS+1-KTR1
73      CONTINUE
        XP1(I) = XBAS1(IK)
        YP1(I) = YBAS1(IK)
        ZP1(I) = ZBAS1(IK)
        U1(I) = 0.
        V1(I) = 0.
        W1(I) = 0.
70      CONTINUE
        DO 80 I=1,KSEG
          IF (I-1) 81,81,82
81        IK = 1
          GO TO 83
82        IK = (I-1)*NPS+1-KTR2
83        CONTINUE
          XP2(I) = XBAS2(IK)
          YP2(I) = YBAS2(IK)
          ZP2(I) = ZBAS2(IK)
          U2(I) = 0.
          V2(I) = 0.
          W2(I) = 0.
80        CONTINUE
          PAR(5) = F1
          DO 110 J=1,KSEG
          DO 110 I=1,KSEG
          PAR(6) = VELJ1(I)
          PAR(9) = DIR1(I)

```

```

      IF (I-1) 111,112,111
112  NF = NPS+1-KTR1
      GO TO 115
111  IF (I-KSEG) 113,114,114
113  NF = NPS+1
      GO TO 115
114  NF = NL
      IF (NL-2) 110,110,115
115  CONTINUE
      CALL VELOC (1,NF,Z1(1,I),X1(1,I),DXDZ1(1,I),UJ1(1,I),D1(1,I),
1  UUE1(1,I),XJ1(I),YJ1(I),ZJ1(I),DJET1(I),CF1(1,1,I),PAR,XP2(J),
2  YP2(J),ZP2(J),UIND,VIND,WIND,SDXDZ1(1,I))
      U2(J) = U2(J)+UIND
      V2(J) = V2(J)+VIND
      W2(J) = W2(J)+WIND
110  CONTINUE
      IF (IHOLD1-1) 150,150,120
120  PAR(6) = VELJ3
      PAR(5) = F3
      PAR(9) = DR3
      N3 = ITHR+1
      DO 121 J=1,KSEG
      CALL VELOC (1,N3,Z3,X3,DXDZ3,UJ3,D3,UUE3,XJ3,YJ3,ZJ3,DJET3,
1  CF3,PAR,XP2(J),YP2(J),ZP2(J),UIND,VIND,WIND,SDXDZ3)
      U2(J) = U2(J)+UIND
      V2(J) = V2(J)+VIND
      W2(J) = W2(J)+WIND
121  CONTINUE
      PAR(6) = VELJ2T
      PAR(5) = F2
      PAR(9) = DIR2T
      DO 122 J=1,KSEG
      CALL VELOC (1,N3,Z2T,X2T,DXDZ2T,UJ2T,D2T,UUE2T,XJ2T,YJ2T,ZJ2T,
1  DJET2T,CF2T,PAR,XP2(J),YP2(J),ZP2(J),UIND,VIND,WIND,SDXDZ2T)
      U2(J) = U2(J)-UIND
      V2(J) = V2(J)-VIND
      W2(J) = W2(J)-WIND
122  CONTINUE
150  CONTINUE
      PAR(5) = F2
      DO 210 J=1,KSEG
      DO 210 I=1,KSEG
      PAR(6) = VELJ2(I)
      PAR(9) = DIR2(I)
      IF (I-1) 211,212,211
212  NF = NPS+1-KTR2
      GO TO 215
211  IF (I-KSEG) 213,214,214
213  NF = NPS+1
      GO TO 215
214  NF = NL
      IF (NL-2) 210,210,215
215  CONTINUE

```

```

      CALL VELOC (1,NF,Z2(1,I),X2(1,I),DXDZ2(1,I),UJ2(1,I),D2(1,I),
1 UUE2(1,I),XJ2(1,I),YJ2(1,I),ZJ2(1,I),DJET2(1,I),CF2(1,1,I),PAR,XP1(J),
2 YP1(J),ZP1(J),UIND,VIND,WIND,SDXDZ2(1,I))
      U1(J) = U1(J)+UIND
      V1(J) = V1(J)+VIND
      W1(J) = W1(J)+WIND
210  CONTINUE
      IF (IHOLD1-1) 250,250,220
220  PAR(6) = VELJ3
      PAR(5) = F3
      PAR(9) = DR3
      DO 221 J=1,KSEG
      CALL VELOC (1,N3,Z3,X3,DXDZ3,UJ3,D3,UUE3,XJ3,YJ3,ZJ3,DJET3,
1 CF3,PAR,XP1(J),YP1(J),ZP1(J),UIND,VIND,WIND,SDXDZ3)
      U1(J) = U1(J)+UIND
      V1(J) = V1(J)+VIND
      W1(J) = W1(J)+WIND
221  CONTINUE
      PAR(6) = VELJ1T
      PAR(5) = F1
      PAR(9) = DIR1T
      DO 222 J=1,KSEG
      CALL VELOC (1,N3,Z1T,X1T,DXDZ1T,UJ1T,D1T,UUE1T,XJ1T,YJ1T,ZJ1T,
1 DJET1T,CF1T,PAR,XP1(J),YP1(J),ZP1(J),UIND,VIND,WIND,SDXDZ1T)
      U1(J) = U1(J)-UIND
      V1(J) = V1(J)-VIND
      W1(J) = W1(J)-WIND
222  CONTINUE
250  CONTINUE
      RETURN
      END

```

#### SUBROUTINE BITEST (I,TNEG,KS)

C  
C  
C  
C

TESTS FOR BLOCKAGE AND INTERSECTION, CALLED AS PART OF INTEGRATION  
LOOP

```

      DIMENSION X1(11,10),Z1(11,10),UJ1(11,10),D1(11,10),DXDZ1(11,10)
      DIMENSION X2(11,10),Z2(11,10),UJ2(11,10),D2(11,10),DXDZ2(11,10)
      DIMENSION X3(100),Z3(100),UJ3(100),D3(100),DXDZ3(100)
      DIMENSION XBAS1(100),YBAS1(100),ZBAS1(100)
      DIMENSION XBAS2(100),YBAS2(100),ZBAS2(100)
      DIMENSION XBAS3(100),YBAS3(100),ZBAS3(100)
      DIMENSION CF1(3,3,10),CF2(3,3,10),CF1T(3,3),CF2T(3,3),CF3(3,3)
      DIMENSION UUE1(11,10),UUE2(11,10),UUE1T(100),UUE2T(100),UUE3(100)
      DIMENSION PAR(15)
      DIMENSION XJ1(10),YJ1(10),ZJ1(10),DJET1(10),VELJ1(10)
      DIMENSION XJ2(10),YJ2(10),ZJ2(10),DJET2(10),VELJ2(10)
      DIMENSION ALFQ1(10),BETQ1(10),GETQ1(10),ALFQ2(10),BETQ2(10),
1 GETQ2(10)
      DIMENSION DIR1(10),DIR2(10),ZS01(10),ZS02(10),UFACT1(10),
1 UFACT2(10)

```

```

COMMON/BLK1/CF1,CF2,CF1T,CF2T,CF3,UUE1,UUE2,UUE1T,UUE2T,UUE3,PAR
COMMON/BLK2/X1,Z1,UJ1,D1,DXDZ1,X2,Z2,UJ2,D2,DXDZ2
COMMON/BLK4/X3,Z3,UJ3,D3,DXDZ3
COMMON/BLK5/XBAS1,YBAS1,ZBAS1,XBAS2,YBAS2,ZBAS2,XBAS3,YBAS3,ZBAS3
COMMON/BLK7/ALFQ,BETQ,GETQ,F1,F2,F3,VKONST
COMMON/BLK8/ALFQ1,BETQ1,GETQ1,ALFQ2,BETQ2,GETQ2
COMMON/BLK9/MULT,IHOLD1,KOUNT1,IONE,ITWO,ITHR,N1,N2,N3,IFIX1
COMMON/BLK10/XJ1,YJ1,ZJ1,DJET1,VELJ1,XJ2,YJ2,ZJ2,DJET2,VELJ2
COMMON/BLK12/XJ3,YJ3,ZJ3,DJET3,VELJ3
COMMON/BLK13/G,G2,G3,STEP1,STEP12,STEP13
COMMON/BLK14/V2X1,V2Y1,V2Z1,V2X2,V2Y2,V2Z2
COMMON/BLK15/DIR1,DIR2,DIR1T,DIR2T,DR3,ZS01,ZS02,UFACT1,UFACT2
COMMON/BLK17/GS,A,B,C,IS1,IS2,NPS

```

C

```

DE = .0001*DJET1(1)
IF (MULT-2) 21,200,200
200 IF (IHOLD1-1) 201,202,21
201 IF (TNEG) 203,203,204
203 CALL XPROD (V2X1,V2Y1,V2Z1,ALFQ1(KS),BETQ1(KS),GETQ1(KS),XT1,
1 YT1,ZT1)
CALL XPROD (XT1,YT1,ZT1,ALFQ1(KS),BETQ1(KS),GETQ1(KS),CFNX,
1 CFNY,CFNZ)
CALL PLANE (CFNX,CFNY,CFNZ,XBAS1(I),YBAS1(I),ZBAS1(I),V2X2,V2Y2,
1 V2Z2,XJ2(KS),YJ2(KS),ZJ2(KS),XINT,YINT,ZINT)
IF (YINT-YJ2(KS)-DE) 205,205,22
204 UUE2(I,KS) = 1.*UFACT2(KS)
CALL XPROD (V2X2,V2Y2,V2Z2,ALFQ2(KS),BETQ2(KS),GETQ2(KS),XT2,
1 YT2,ZT2)
CALL XPROD (XT2,YT2,ZT2,ALFQ2(KS),BETQ2(KS),GETQ2(KS),CFNX,
1 CFNY,CFNZ)
CALL PLANE (CFNX,CFNY,CFNZ,XBAS2(I),YBAS2(I),ZBAS2(I),V2X1,V2Y1,
1 V2Z1,XJ1(KS),YJ1(KS),ZJ1(KS),XINT,YINT,ZINT)
IF (YINT-YJ1(KS)-DE) 205,205,22
205 IHOLD1 = 1
202 IF (TNEG) 206,206,207
206 ITWO = I-KOUNT1
GO TO 208
207 IONE = I-KOUNT1
208 IT1 = IONE
IT2 = ITWO
N1 = IT1+1
N2 = IT2+1
IF (KS-1) 210,210,211
210 IS1 = IONE
IS2 = ITWO
GO TO 212
211 IS1 = I-(KS-1)*NPS
IS2 = IS1
212 CONTINUE
CALL COMP (V2X1,V2Y1,V2Z1,V2X2,V2Y2,V2Z2,XBAS1(IT1),YBAS1(IT1),
1 ZBAS1(IT1),XBAS2(IT2),YBAS2(IT2),ZBAS2(IT2),Z1(IS1,KS),
2 Z2(IS2,KS),D1(IS1,KS),DJET1(KS),D2(IS2,KS),DJET2(KS),VELJ1(KS),
3 VELJ2(KS),DXDZ1(IS1,KS),KS,UUE2(IS2,KS),A1,A2,DR3,INT)
IF (INT) 21,21,209
209 IHOLD1 = 2

```

```

N1 = IT1
N2 = IT2
IFIX1 = I
CALL BALANC (XBAS1(IT1),YBAS1(IT1),ZBAS1(IT1),XBAS2(IT2),
1 YBAS2(IT2),ZBAS2(IT2),UJ1(IS1,KS),UJ2(IS2,KS),VELJ1(KS),
2 VELJ2(KS),A1,A2,V2X1,V2Y1,V2Z1,V2X2,V2Y2,V2Z2,DR3,XJ3,YJ3,ZJ3,
3 DJET3,V2X3,V2Y3,V2Z3,VELJ3)
CALL CFCAL (ALFQ,BETQ,GETQ,V2X3,V2Y3,V2Z3,CF3)
CALL ROTATE (V2X3,V2Y3,V2Z3,CF3,VXT,VYT,VZT,0)
UJ3(1) = 1.
D3(1) = 1.
X3(1) = 0.
Z3(1) = 0.
DXDZ3(1) = VXT/VZT
XBAS3(1) = XJ3
YBAS3(1) = YJ3
ZBAS3(1) = ZJ3
D = ATAN(VXT/VZT)
IF (VXT) 901,902,902
901 F3 = .3*COS(D)
GO TO 903
902 F3 = .3/COS(D)
903 CONTINUE
G3 = GS*DJET1(1)/DJET3
STEPI3 = .2*G3
GO TO 21
22 KOUNT1 = KOUNT1+1
IS1 = I
IS2 = I
21 CONTINUE
RETURN
END

```

#### SUBROUTINE INTEG (I,TNEG,KS)

C  
C  
C

INTEGRATION OF THE EQUATIONS OF MOTION FOR THE JET PATH

EXTERNAL DERIV

C

```

DIMENSION X1(11,10),Z1(11,10),UJ1(11,10),D1(11,10),DXDZ1(11,10)
DIMENSION X2(11,10),Z2(11,10),UJ2(11,10),D2(11,10),DXDZ2(11,10)
DIMENSION X3(100),Z3(100),UJ3(100),D3(100),DXDZ3(100)
DIMENSION XBAS1(100),YBAS1(100),ZBAS1(100)
DIMENSION XBAS2(100),YBAS2(100),ZBAS2(100)
DIMENSION XBAS3(100),YBAS3(100),ZBAS3(100)
DIMENSION CF1(3,3,10),CF2(3,3,10),CF1T(3,3),CF2T(3,3),CF3(3,3)
DIMENSION UUE1(11,10),UUE2(11,10),UUE1T(100),UUE2T(100),UUE3(100)
DIMENSION SDXDZ1(11,10),SDXDZ2(11,10),SXZ1T(100),SXZ2T(100),
1 SDXDZ3(100)
DIMENSION PAR(15)
DIMENSION XJ1(10),YJ1(10),ZJ1(10),DJET1(10),VELJ1(10)
DIMENSION XJ2(10),YJ2(10),ZJ2(10),DJET2(10),VELJ2(10)

```

```

    DIMENSION ALFQ1(10),BETQ1(10),GETQ1(10),ALFQ2(10),BETQ2(10),
1 GETQ2(10)
    DIMENSION DIR1(10),DIR2(10),ZS01(10),ZS02(10),UFACT1(10),
1 UFACT2(10)

```

C

```

COMMON/BLK1/CF1,CF2,CF1T,CF2T,CF3,UUE1,UUE2,UUE1T,UUE2T,UUE3,PAR
COMMON/BLK2/X1,Z1,UJ1,D1,DXDZ1,X2,Z2,UJ2,D2,DXDZ2
COMMON/BLK4/X3,Z3,UJ3,D3,DXDZ3
COMMON/BLK5/XBAS1,YBAS1,ZBAS1,XBAS2,YBAS2,ZBAS2,XBAS3,YBAS3,ZBAS3
COMMON/BLK7/ALFQ,BETQ,GETQ,F1,F2,F3,VKONST
COMMON/BLK8/ALFQ1,BETQ1,GETQ1,ALFQ2,BETQ2,GETQ2
COMMON/BLK9/MULT,IHOLD1,KOUNT1,IONE,ITWO,ITHR,N1,N2,N3,IFIX1
COMMON/BLK10/XJ1,YJ1,ZJ1,DJET1,VELJ1,XJ2,YJ2,ZJ2,DJET2,VELJ2
COMMON/BLK12/XJ3,YJ3,ZJ3,DJET3,VELJ3
COMMON/BLK13/G,G2,G3,STEPI,STEPI2,STEPI3
COMMON/BLK14/V2X1,V2Y1,V2Z1,V2X2,V2Y2,V2Z2
COMMON/BLK15/DIR1,DIR2,DIR1T,DIR2T,DR3,ZS01,ZS02,UFACT1,UFACT2
COMMON/BLK16/SDXDZ1,SDXDZ2,SDXDZ3, SXZ1T, SXZ2T
COMMON/BLK17/GS,A,B,C,IS1,IS2,NPS

```

C

```

    DIMENSION FIN(4),FOUT(4)

```

C

```

    IF (MULT-2) 24,51,51
51 IF (IHOLD1-2) 25,30,30
25 IF (TNEG) 24,24,27
27 IF (IHOLD1) 28,28,24
24 UUE1(IS1,KS) = 1.*UFACT1(KS)
    PAR(6) = VELJ1(KS)*UUE1(IS1,KS)
    PAR(5) = F1
    PAR(9) = DIR1(KS)
    PAR(15) = 1.
    Z1(IS1+1,KS) = Z1(IS1,KS)+G
    FIN(1) = UJ1(IS1,KS)
    FIN(2) = D1(IS1,KS)
    FIN(3) = X1(IS1,KS)
    FIN(4) = DXDZ1(IS1,KS)
    CALL ADAMS (4,Z1(IS1,KS),Z1(IS1+1,KS),STEPI,G,999,1.0E-04,1.0E-05,
1 0,FIN,FOUT,PAR,DERIV)
    UJ1(IS1+1,KS) = FOUT(1)
    D1(IS1+1,KS) = FOUT(2)
    X1(IS1+1,KS) = FOUT(3)
    DXDZ1(IS1+1,KS) = FOUT(4)
    SDXDZ1(IS1+1,KS) = PAR(10)
    CALL OUTPT1(X1(IS1+1,KS),Z1(IS1+1,KS),DXDZ1(IS1+1,KS),CF1,KS,
1 DJET1(KS),XJ1(KS),YJ1(KS),ZJ1(KS),XBAS1(IONE+1),YBAS1(IONE+1),
2 ZBAS1(IONE+1),V2X1,V2Y1,V2Z1)
    IF (MULT-2) 50,41,41
41 IF (IHOLD1) 50,50,28
28 PAR(6) = VELJ2(KS)*UUE2(IS2,KS)
    PAR(5) = F2
    PAR(9) = DIR2(KS)
    PAR(15) = 2.
    Z2(IS2+1,KS) = Z2(IS2,KS)+G2
    FIN(1) = UJ2(IS2,KS)

```

```

    FIN(2) = D2(IS2,KS)
    FIN(3) = X2(IS2,KS)
    FIN(4) = DXDZ2(IS2,KS)
    CALL ADAMS (4,Z2(IS2,KS),Z2(IS2+1,KS),STEP12,G2,999,1.0E-04,
1 1.0E-05,0,FIN,FOUT,PAR,DERIV)
    UJ2(IS2+1,KS) = FOUT(1)
    D2(IS2+1,KS) = FOUT(2)
    X2(IS2+1,KS) = FOUT(3)
    DXDZ2(IS2+1,KS) = FOUT(4)
    SDXDZ2(IS2+1,KS) = PAR(10)
    CALL OUTPT1(X2(IS2+1,KS),Z2(IS2+1,KS),DXDZ2(IS2+1,KS),CF2,KS,
1 DJET2(KS),XJ2(KS),YJ2(KS),ZJ2(KS),XBAS2(ITWO+1),YBAS2(ITWO+1),
2 ZBAS2(ITWO+1),V2X2,V2Y2,V2Z2)
    GO TO 50
30  ITHR = I-IFIX1+1
    UUE3(ITHR) = 1.
    PAR(6) = VELJ3
    PAR(5) = F3
    PAR(9) = DR3
    PAR(15) = 3.
    Z3(ITHR+1) = Z3(ITHR)+G3
    FIN(1) = UJ3(ITHR)
    FIN(2) = D3(ITHR)
    FIN(3) = X3(ITHR)
    FIN(4) = DXDZ3(ITHR)
    CALL ADAMS(4,Z3(ITHR),Z3(ITHR+1),STEP13,G3,999,1.0E-04,
1 1.0E-05,0,FIN,FOUT,PAR,DERIV)
    UJ3(ITHR+1) = FOUT(1)
    D3(ITHR+1) = FOUT(2)
    X3(ITHR+1) = FOUT(3)
    DXDZ3(ITHR+1) = FOUT(4)
    SDXDZ3(ITHR+1) = PAR(10)
    CALL OUTPT (X3(ITHR+1),Z3(ITHR+1),DXDZ3(ITHR+1),CF3,DJET3,XJ3,YJ3,
1 ZJ3,XBAS3(ITHR+1),YBAS3(ITHR+1),ZBAS3(ITHR+1),V2X3,V2Y3,V2Z3)
50  CONTINUE
    RETURN
    END

```

#### SUBROUTINE DERIV (Z,FN,FPR,PAR)

```

C
C
C  COMPUTES DERIVATIVES FOR ADAMS PREDICTOR/CORRECTOR METHOD
C
C  DIMENSION FN(1),FPR(1),PAR(1)

E1 = PAR(1)
E2 = PAR(2)
E3 = PAR(3)
F = PAR(5)
VELJ=PAR(6)
PI = PAR(7)
C1 = PAR(8)
DR = PAR(9)
UJ = FN(1)

```



```

D = FN(2)
DXDZ=FN(4)
JET = PAR(15)
IF (JET-2) 1,2,5
1 A = PAR(11)
  B = PAR(12)
  GO TO 3
2 A = PAR(13)
  B = PAR(14)
3 CONTINUE
  IF (A) 5,5,4
4 IF (Z-A/B) 6,6,7
6 AN1 = (A*A-2.*A*B*Z+B*B*Z*Z)/(D*D)
  AN2 = PI*UJ*(A*B-B*B*Z)/2.
  AN3 = 1.-4.*AN1
  GO TO 10
7 IF (JET-1) 8,8,9
8 PAR(11) = 0.
  GO TO 5
9 PAR(13) = 0.
5 AN1 = 0.
  AN2 = 0.
  AN3 = 1.
10 CONTINUE
  COST = 1./SQRT(1.+DXDZ*DXDZ)
  SINT = SIGN(1.,DXDZ)*SQRT(1.-COST*COST)
  E = E2/(1.+E3*COST/(VELJ*UJ))
  IF (VELJ*UJ-SINT) 11,12,12
11 E = 0.
12 ZS0 = (1.-DR)*VELJ*F/.75
  ZP = Z+ZS0
  IF (ZP-VELJ*F) 47,60,60
47 IF (ZP-10.) 40,60,60
40 IF (ZP-.6*VELJ*F) 42,43,43
42 E = E*.1/.32
  GO TO 60
43 IF (ZP-.8*VELJ*F) 44,45,45
44 E = E*.12/.32
  GO TO 60
45 E = E*.21/.32
60 ZOVM = ZP/VELJ
  IF (ZOVM-F) 22,23,23
22 VAR = SQRT((1.+(1.-.75*ZOVM/F)**2)/2.)
  XT = 1.-.75*ZOVM/F
  XT = 1./XT
  CD = (-XT*XT+6.6*XT+.4)/6.
  VAR1 = E1*COST+E*(VELJ*UJ-SINT)*PI*VAR
  VAR2 = VELJ*VELJ*COST
  VAR3 = .25*PI*(1.-.75*ZOVM/F-AN1)*UJ*D
  VARA = .25*PI*(1.-.75*ZOVM/F)*UJ*D
  DUJ = (VAR1*SINT/VAR2-VAR1*UJ/(VELJ*COST))/VAR3
  DD = (VAR1*D/(VELJ*COST)+3.*PI*D*D*UJ/(16.*F*VELJ)-VAR3*D*DUJ/
1 UJ-AN2)/(2.*VARA)
  VAR4 = (E1+.5*CD)*COST+E*(VELJ*UJ-SINT)*PI*VAR

```

```

DDXDZ= VAR4/(VAR2*COST*VAR3*UJ)
GO TO 15
23 VAR1 = E1*COST+E*(VELJ*UJ-SINT)*C1
CD = 1.8
DUJ = 16.*VAR1*(SINT/(VELJ*VELJ*COST)-UJ/(VELJ*COST))/(PI*D*UJ)
DUJ = DUJ/AN3
DD = 8.*(VAR1/(VELJ*COST)-AN3*PI*D*DUJ/16.-AN2/D)/(PI*UJ)
VAR4 = (E1+.5*CD)*COST+E*(VELJ*UJ-SINT)*C1
DDXDZ= 16.*VAR4/(PI*VELJ*VELJ*D*UJ*UJ*COST*COST)
DDXDZ = DDXDZ/AN3
15 CONTINUE
PAR(10) = DDXDZ
FPR(1) = DUJ
FPR(2) = DD
FPR(3) = DXDZ
FPR(4) = DDXDZ
RETURN
END

```

```

SUBROUTINE VELOC (N1,N2,Z,X,DXDZ,UJ,D,UUE,XJ,YJ,ZJ,DJET,CF,PAR,
1 XO,YO,ZO,UIF,VIF,WIF,D2XDZ2)

```

```

C EVALUATES INDUCED VELOCITIES AT ONE CONTROL POINT (XO,YO,ZO IN
C FIXED COORDINATE SYSTEM) FOR A GIVEN JET
C

```

```

C COMMON/BLK19/DIARAT,DREF

```

```

C DIMENSION Z(1),X(1),DXDZ(1),UJ(1),D(1),UUE(1),PAR(1)
C DIMENSION CF(3,3)
C DIMENSION D2XDZ2(1)

```

```

C E2 = PAR(2)
E3 = PAR(3)
F = PAR(5)
VELJ=PAR(6)
PI = PAR(7)
C1 = PAR(8)
DR = PAR(9)
N = N2-N1+1
IF (N/2-(N+1)/2) 1,2,2
1 M = (N-1)/2
GO TO 3
2 M = (N-2)/2
3 XPT = (XO-XJ)/DJET
YPT = (YO-YJ)/DJET
ZPT = (ZO-ZJ)/DJET
CALL ROTATE (XPT,YPT,ZPT,CF,A,B,C,0)
UI = 0.
VI = 0.
WI = 0.
M1 = M+1
DO 21 K=N1,M1
E1 = PAR(1)

```

```

      IF (K-M) 11,11,10
10  IF (N/2-(N+1)/2) 22,12,12
12  I = 2*K-1
      ZINCR = Z(I+1)-Z(I)
      GO TO 14
11  I = 2*K
      ZINCR = Z(I+1)-Z(I-1)
14  COST = 1./SQRT(1.+DXDZ(I)*DXDZ(I))
      SINT = SIGN(1.,DXDZ(I))*SQRT(1.-COST*COST)
      SIE = -((Z(I)-C)*COST+(X(I)-A)*SINT)
      ETA = B
      ZETA = (Z(I)-C)*SINT-(X(I)-A)*COST
      D1 = .5*D(I)
      DOUB1 = SIE*SIE+ETA*ETA+ZETA*ZETA
      DOUB2 = SQRT(DOUB1)
      UBLOCK = .5*D1*D1*ZINCR*COST*(1.-3.*ZETA*ZETA/DOUB1)/(DOUB1*DOUB2)
1  -SINT*1.5*SIE*ZETA*D1*D1*ZINCR/(DOUB1*DOUB1*DOUB2)
      VBLOCK = -1.5*ZETA*ETA*D1*D1*ZINCR/(DOUB1*DOUB1*DOUB2)
      WBLOCK = -.5*D1*D1*ZINCR*SINT*(1.-3.*ZETA*ZETA/DOUB1)/(DOUB1*
1  DOUB2)-COST*1.5*SIE*ZETA*D1*D1*ZINCR/(DOUB1*DOUB1*DOUB2)
      VELJE = VELJ*UUE(I)
      CURV = D2XDZ2(I)/((1.+DXDZ(I)*DXDZ(I))**.5)
      CURV = 3.*CURV*DREF/DJET
      E1 = E1-CURV/COST
      E = E2/(1.+E3*COST/(VELJE*UJ(I)))
      IF (VELJE*UJ(I)-SINT) 51,52,52
51  E = 0.
52  ZS0 = (1.-DR)*VELJE*F/.75
      ZP = Z(I)+ZS0
      IF (ZP-VELJE*F) 47,60,60
47  IF (ZP-10.) 40,60,60
40  IF (ZP-.6*VELJE*F) 42,43,43
42  E = E*.1/.32
      GO TO 60
43  IF (ZP-.8*VELJE*F) 44,45,45
44  E = E*.12/.32
      GO TO 60
45  E = E*.21/.32
60  ZOVM = ZP/VELJE
      IF (ZOVM-F) 31,32,32
31  VARB = (1.-.375*ZOVM/F)
      VAR = SQRT((1.+(1.-.75*ZOVM/F)**2)/2.)
      HT3 = .25*ZINCR*(E1+E*PI*VAR*(VELJE*UJ(I)-SINT)/COST)
      GO TO 33
32  VARB = .625
      HT3 = .25*ZINCR*(E1+E*(VELJE*UJ(I)-SINT)*C1/COST)
33  UBLOCK = UBLOCK*VARB
      VBLOCK = VBLOCK*VARB
      WBLOCK = WBLOCK*VARB
      Z1 = (C-Z(I))*(C-Z(I))+(A-X(I))*(A-X(I))
      Z2 = SQRT((B-D1)*(B-D1)+Z1)
      Z3 = SQRT((B+D1)*(B+D1)+Z1)
      USINK = -HT3*(X(I)-A)*((B-D1)/(Z1*Z2)-(B+D1)/(Z1*Z3))/PI
      VSINK = -HT3*(1./Z2-1./Z3)/PI

```

```

        WSINK = -HT3*(Z(I)-C)*((B-D1)/(Z1*Z2)-(B+D1)/(Z1*Z3))/PI
        IF (UUE(I)-1.) 6,5,6
6      FACT = 1./UUE(I)
        UBLOCK = UBLOCK*FACT
        VBLOCK = VBLOCK*FACT
        WBLOCK = WBLOCK*FACT
        USINK = USINK*FACT
        VSINK = VSINK*FACT
        WSINK = WSINK*FACT
5      UI = UI+USINK+UBLOCK
        VI = VI+VSINK+VBLOCK
21     WI = WI+WSINK+WBLOCK
22     CALL ROTATE (UIF,VIF,WIF,CF,UI,VI,WI,1)
691    FORMAT (6F12.5)
        RETURN
        END

```

```

        SUBROUTINE COMP(VX1,VY1,VZ1,VX2,VY2,VZ2,X1,Y1,Z1,X2,Y2,Z2,Z1L,Z2L,
1      D1,DJ1,D2,DJ2,V1,V2,SL1,KS,UUEFF,A1,A2,DRAT,IND)
C
C      COMPUTES U/UEFFECTIVE AND TESTS FOR INTERSECTION OF CENTERLINES
C
        DIMENSION DIR1(10),DIR2(10),ZS01(10),ZS02(10),UFACT1(10),
1      UFACT2(10)
C
        COMMON/BLK7/ALFQ,BETQ,GETQ,F1,F2,F3,VKONST
        COMMON/BLK15/DIR1,DIR2,DIR1T,DIR2T,DR3,ZS01,ZS02,UFACT1,UFACT2
        COMMON/BLK19/DIARAT,DREF
C
        IND = 0
        PI = 3.1416
        CALL XPROD (VX1,VY1,VZ1,ALFQ,BETQ,GETQ,CFNX,CFNY,CFNZ)
        CALL XPROD (VX2,VY2,VZ2,ALFQ,BETQ,GETQ,XT2,YT2,ZT2)
        CALL PLANE (CFNX,CFNY,CFNZ,X1,Y1,Z1,XT2,YT2,ZT2,X2,Y2,Z2,XI,YI,ZI)
        DIST = SQRT((XI-X2)**2+(YI-Y2)**2+(ZI-Z2)**2)
C
C      COMPUTE U/UEFFECTIVE
C
        R = D1*DJ1*.5-DIST
        FACT = (1.0+R/(D2*DJ2*.5))*0.5
        IF (FACT-1.) 10,10,11
11     UUEFF = VKONST
        GO TO 15
10     IF (FACT) 13,13,12
13     UUEFF = 1.
        GO TO 15
12     TEST1 = D1*DJ1*.5+DIST
        TEST2 = D2*DJ2*.5
        IF (TEST1-TEST2) 16,14,14
16     FACT = D1*DJ1/(D2*DJ2)
14     UEFU = 1.+(1./VKONST-1.)*FACT
        UUEFF = 1./UEFU
15     CONTINUE

```

UUEFF = UUEFF\*UFACT2(KS)

TEST FOR INTERSECTION OF CENTERLINES

COST = 1./SQRT(1.+SL1\*SL1)

SUMD = DJ1\*D1\*.5

IF (DIST-SUMD) 22,99,99

22 DISTN = SQRT((X1-XI)\*\*2+(Y1-YI)\*\*2+(Z1-ZI)\*\*2)

IF (DIR1(KS)-.2501) 25,25,51

51 ZOVM = (ZS01(KS)+Z1L)/(V1\*UFACT1(KS))

IF (ZOVM-F1) 24,24,25

24 FACT1 = 1.-.75\*ZOVM/F1

GO TO 26

25 FACT1 = .25

26 IF (DIR2(KS)-.2501) 28,28,52

52 ZOVM = (ZS02(KS)+Z2L)/(V2\*UUEFF)

IF (ZOVM-F2) 27,27,28

27 FACT2 = 1.-.75\*ZOVM/F2

GO TO 29

28 FACT2 = .25

29 SUMD = DJ1\*D1\*FACT1\*COST\*.5

IF (DISTN-SUMD) 30,30,40

30 IND = 1

GO TO 45

40 IF (X2-X1) 30,30,99

45 A1 = PI\*FACT1\*D1\*D1\*DJ1\*DJ1\*.25

A2 = PI\*FACT2\*D2\*D2\*DJ2\*DJ2\*.25

DRAT = DIARAT

99 CONTINUE

RETURN

END

SUBROUTINE VEL1 (MULT,ALFA,VK1)

COMPUTES EFFECTIVE VELOCITY RATIO FOR DOWNSTREAM JET AT EXIT

DIMENSION XJ1(10),YJ1(10),ZJ1(10),DJET1(10),VELJ1(10)

DIMENSION XJ2(10),YJ2(10),ZJ2(10),DJET2(10),VELJ2(10)

COMMON/BLK7/ALFQ,BETQ,GETQ,F1,F2,F3,VKONST

COMMON/BLK10/XJ1,YJ1,ZJ1,DJET1,VELJ1,XJ2,YJ2,ZJ2,DJET2,VELJ2

COMMON/BLK14/V2X1,V2Y1,V2Z1,V2X2,V2Y2,V2Z2

VELJ1(1) = 1./VELJ1(1)

IF (MULT-2) 15,1,1

1 VELJ2(1) = 1./VELJ2(1)

DOTP = (XJ2(1)-XJ1(1))\*ALFQ+(YJ2(1)-YJ1(1))\*BETQ+(ZJ2(1)-ZJ1(1))

1 \*GETQ

DEN = SQRT((XJ2(1)-XJ1(1))\*\*2+(YJ2(1)-YJ1(1))\*\*2+(ZJ2(1)-ZJ1(1))

1 \*\*2)

DOTP = DOTP/DEN

IF (ABS(DOTP)-.02) 10,10,11

10 VK1 = 1.

GO TO 15

```

11  CONTINUE
    A = ALFA*.0174533
    ALF = COS(A)
    BET = SIN(A)
    GET = 0.
    CALL XPROD (V2X1,V2Y1,V2Z1,ALF,BET,GET,XT1,YT1,ZT1)
    CALL XPROD (XT1,YT1,ZT1,ALF,BET,GET,CFNX,CFNY,CFNZ)
    CALL PLANE (CFNX,CFNY,CFNZ,XJ1(1),YJ1(1),ZJ1(1),V2X2,V2Y2,V2Z2,
1  XJ2(1),YJ2(1),ZJ2(1),XI,YI,ZI)
    S = SQRT((XJ1(1)-XI)**2+(YJ1(1)-YI)**2+(ZJ1(1)-ZI)**2)/DJET1(1)
    VK1 = (S+.75)/(S-1.)
    DOTP = V2X1*ALFQ+V2Y1*BETQ+V2Z1*GETQ
    AIN = ACOS(DOTP)
    IF (DOTP) 4,4,5
4  AIN = AIN-3.14159/2.
    GO TO 6
5  AIN = 3.14159/2.-AIN
6  CONTINUE
    SIN2 = SIN(AIN)*SIN(AIN)
    COS2 = COS(AIN)*COS(AIN)
    C = 1./VK1
    VK1 = 1./SQRT(SIN2+C*C*COS2)
15 CONTINUE
    RETURN
    END

```

```

SUBROUTINE BALANC (X1,Y1,Z1,X2,Y2,Z2,UJ1,UJ2,V1,V2,A1,A2,VX1,VY1,
1  VZ1,VX2,VY2,VZ2,FACT1,X3,Y3,Z3,DJ3,VX3,VY3,VZ3,
2  VELJ3)

```

ESTABLISHES INITIAL CONDITIONS FOR NEW JET FROM MOMENTUM BALANCE

```

PI = 3.1416
X3 = (X1+X2)*.5
Y3 = (Y1+Y2)*.5
Z3 = (Z1+Z2)*.5
XM1 = UJ1*V1*A1
XM2 = UJ2*V2*A2
DEN = XM1+XM2
UJX = (XM1*UJ1*V1*VX1+XM2*UJ2*V2*VX2)/DEN
UJY = (XM1*UJ1*V1*VY1+XM2*UJ2*V2*VY2)/DEN
UJZ = (XM1*UJ1*V1*VZ1+XM2*UJ2*V2*VZ2)/DEN
VELJ3 = SQRT (UJX*UJX+UJY*UJY+UJZ*UJZ)
VX3 = UJX/VELJ3
VY3 = UJY/VELJ3
VZ3 = UJZ/VELJ3
A3 = DEN/VELJ3
DJ3 = SQRT (4.*A3/(PI*FACT1))
RETURN
END

```

SUBROUTINE FIX (UA,VA,WA,UB,VB,WB,NO)

LIMITS MUTUALLY INDUCED VELOCITIES TO A MAXIMUM VALUE

DIMENSION UA(1),VA(1),WA(1),UB(1),VB(1),WB(1)

DEL = .6

DO 5 I=1,NO

IF (ABS(UA(I))-DEL) 1,1,3

1 IF (ABS(VA(I))-DEL) 2,2,3

2 IF (ABS(WA(I))-DEL) 5,5,3

3 UA(I) = UA(I-1)

VA(I) = VA(I-1)

WA(I) = WA(I-1)

5 CONTINUE

DO 15 I=1,NO

IF (ABS(UB(I))-DEL) 11,11,13

11 IF (ABS(VB(I))-DEL) 12,12,13

12 IF (ABS(WB(I))-DEL) 15,15,13

13 UB(I) = UB(I-1)

VB(I) = VB(I-1)

WB(I) = WB(I-1)

15 CONTINUE

RETURN

END

SUBROUTINE MUINT (ALF,BET,GET,U,V,W,ALFM,BETM,GETM,UUIND)

COMPUTES DIRECTION COSINES OF MODIFIED FREESTREAM AND THE  
MODIFIED FREESTREAM/FREESTREAM VELOCITY RATIO

A = ALF+U

B = BET+V

G = GET+W

D = SQRT(A\*\*2+B\*\*2+G\*\*2)

ALFM = A/D

BETM = B/D

GETM = G/D

UUIND = 1./D

RETURN

END

SUBROUTINE PRTOU (IGEOM,XO,YO,ZO,U,V,W,CP,NK,ITR,ITF,OV)

PRINTS OUT COMPUTED ANSWERS, INFORMATION INCLUDES JET CENTERLINE  
DATA AND INDUCED VELOCITIES AT CONTROL POINTS

DIMENSION X1T(100),Z1T(100),UJ1T(100),D1T(100),DXDZ1T(100)

DIMENSION X2T(100),Z2T(100),UJ2T(100),D2T(100),DXDZ2T(100)

DIMENSION X3(100),Z3(100),UJ3(100),D3(100),DXDZ3(100)

DIMENSION XBAS1(100),YBAS1(100),ZBAS1(100)

DIMENSION XBAS2(100),YBAS2(100),ZBAS2(100)

DIMENSION XBAS3(100),YBAS3(100),ZBAS3(100)

```

COMMON/BLK3/X1T,Z1T,UJ1T,D1T,DXDZ1T,X2T,Z2T,UJ2T,D2T,DXDZ2T
COMMON/BLK4/X3,Z3,UJ3,D3,DXDZ3
COMMON/BLK5/XBAS1,YBAS1,ZBAS1,XBAS2,YBAS2,ZBAS2,XBAS3,YBAS3,ZBAS3
COMMON/BLK9/MULT,IHOLD1,KOUNT1,IONE,ITWO,ITHR,N1,N2,N3,IFIX1
COMMON/BLK12/XJ3,YJ3,ZJ3,DJET3,VELJ3

```

```

C
  DIMENSION XO(1),YO(1),ZO(1),U(1),V(1),W(1),CP(1)
C
  WRITE (6,640)
  IF (OV-.01) 15,15,10
10  WRITE (6,601)
601  FORMAT (1H0,/)
  IF (ITR) 5,5,6
  5  WRITE (6,606)
606  FORMAT (1H0,45X,29H*** INITIAL APPROXIMATION ***)
  GO TO 15
  6  IF (ITR-ITF) 7,8,8
  7  WRITE (6,607) ITR
607  FORMAT (1H0,47X,20H*** ITERATION NUMBER,I2,1X,3H***)
  GO TO 15
  8  WRITE (6,608) ITR
608  FORMAT (1H0,39X,20H*** ITERATION NUMBER,I2,21H, FINAL ITERATION **
1*)
  15  WRITE (6,601)
  IF (MULT-2) 1,2,2
  1  WRITE (6,602)
602  FORMAT (1H0,46X,27H** SINGLE JET CENTERLINE **)
  GO TO 20
  2  WRITE (6,603)
603  FORMAT (1H0,43X,33H** CENTERLINES OF JETS 1 AND 2 **)
  IF (IHOLD1-2) 20,4,4
  4  WRITE (6,605)
605  FORMAT (1H ,51X,17HAND COALESCED JET)
  20  CONTINUE
  WRITE (6,630)
630  FORMAT (1H0,42X,35H*****//)
  IF (MULT.GE.1) WRITE (6,610)
  IF (MULT.GE.2) WRITE (6,611)
610  FORMAT (1H0,3X,6HXCkoord,3X,6HYCOORD,3X,6HZCOORD,3X,2HUJ,4X,3HDIA)
611  FORMAT (1H+,42X,6HXCkoord,3X,6HYCOORD,3X,6HZCOORD,3X,2HUJ,4X,3HDIA)
  WRITE (6,612)
612  FORMAT (1H0)
  IF (MULT-2) 30,40,40
  30  CONTINUE
  WRITE (6,616) (XBAS1(I),YBAS1(I),ZBAS1(I),UJ1T(I),D1T(I), I=1,N1)
616  FORMAT (1H ,1X,F8.2,1X,F8.2,1X,F8.2,1X,F5.3,1X,F5.2)
  GO TO 90
  40  IF (N1-N2) 41,42,42
  41  IP1 = N1
  IP2 = N2
  GO TO 43
  42  IP1 = N2
  IP2 = N1
  43  CONTINUE

```



```

      DO 47 I=1,IP1
47  WRITE (6,613) XBAS1(I),YBAS1(I),ZBAS1(I),UJ1T(I),D1T(I),XBAS2(I),
1  YBAS2(I),ZBAS2(I),UJ2T(I),D2T(I)
613  FORMAT (1H ,1X,F8.2,1X,F8.2,1X,F8.2,1X,F5.3,1X,F5.2,1X,F8.2,1X,
1  F8.2,1X,F8.2,1X,F5.3,1X,F5.2,1X,F8.2,1X,F8.2,1X,F8.2,1X,F5.3,1X,
2  F5.2)
      IF (N1-N2) 48,50,44
48  IPP = IP1+1
      DO 45 I=IPP,IP2
45  WRITE (6,614) XBAS2(I),YBAS2(I),ZBAS2(I),UJ2T(I),D2T(I)
614  FORMAT (1H ,40X,F8.2,1X,F8.2,1X,F8.2,1X,F5.3,1X,F5.2,1X,F8.2,1X,
1  F8.2,1X,F8.2,1X,F5.3,1X,F5.2)
      GO TO 50
44  IPP = IP1+1
      DO 46 I=IPP,IP2
46  WRITE (6,613) XBAS1(I),YBAS1(I),ZBAS1(I),UJ1T(I),D1T(I)
50  CONTINUE
      IF (IHOLD1-2) 90,51,51
51  CONTINUE
      V3 = 1./VELJ3
      ZP = YJ3
      YP = -ZJ3
      WRITE (6,615) XJ3,YP,ZP,V3,DJET3
615  FORMAT (1H0,3X,27HPROPERTIES OF COALESCED JET,3X,2HX=,F9.2,3X,2HY=
1  F8.2,3X,2HZ=,F8.2,3X,6HU/UJO=,F5.2,3X,5HD/D0=,F5.2)
      WRITE (6,610)
      WRITE (6,616) (XBAS3(I),YBAS3(I),ZBAS3(I),UJ3(I),D3(I), I=1,N3)
90  CONTINUE
      IF (IGEOM) 200,99,200
200  WRITE (6,640)
640  FORMAT (1H1)
      WRITE (6,635)
635  FORMAT (1H0,38X,44H*** INDUCED VELOCITIES AT CONTROL POINTS ***)
      IF (IGEOM-3) 221,221,222
221  WRITE (6,632)
632  FORMAT (1H0,27X,1HX,8X,1HY,8X,1HZ,12X,1HU,14X,1HV,14X,1HW/)
      WRITE (6,634) (X0(I),Y0(I),Z0(I),U(I),V(I),W(I), I=1,NK)
634  FORMAT (1H ,21X,F9.3,1X,F9.3,1X,F9.3,3E15.5)
      GO TO 99
222  WRITE (6,636)
636  FORMAT (1H ,40X,39HPRESSURE COEFFICIENTS AT CONTROL POINTS)
      WRITE (6,637)
637  FORMAT (1H0,20X,1HX,8X,1HY,8X,1HZ,12X,2HCP,14X,1HU,14X,1HV,14X,
1  1HW/)
      WRITE (6,638) (X0(I),Y0(I),Z0(I),CP(I),U(I),V(I),W(I), I=1,NK)
638  FORMAT (1H ,14X,F9.3,1X,F9.3,1X,F9.3,4E15.5)
99  CONTINUE
      RETURN
      END

```

SUBROUTINE TRANS1 (MULT,ALFA,BETA,PSID)

TRANSFORMS INPUT COORDINATES TO PROGRAM COORDINATES (FIXED)  
CONVERTS ANGLE OF ATTACK AND SIDESLIP TO FRSTRM DIRECTION COS.

DIMENSION XJ1(10),YJ1(10),ZJ1(10),DJET1(10),VELJ1(10)  
DIMENSION XJ2(10),YJ2(10),ZJ2(10),DJET2(10),VELJ2(10)

COMMON/BLK7/ALFQ,BETQ,GETQ,F1,F2,F3,VKONST  
COMMON/BLK10/XJ1,YJ1,ZJ1,DJET1,VELJ1,XJ2,YJ2,ZJ2,DJET2,VELJ2

DIMENSION PSID(1)

A = ALFA\*.0174533

B = BETA\*.0174533

ALFQ = COS(A)\*COS(B)

BETQ = SIN(A)\*COS(B)

GETQ = SIN(B)

YS = YJ1(1)

YJ1(1) = ZJ1(1)

ZJ1(1) = -YS

PSID(1) = -PSID(1)

IF (MULT-2) 5,4,4

4 YS = YJ2(1)

YJ2(1) = ZJ2(1)

ZJ2(1) = -YS

PSID(2) = -PSID(2)

5 CONTINUE

RETURN

END

SUBROUTINE TRANS2 (Y,Z,NO)

TRANSFORMS INPUT COORDINATES TO PROGRAM COORDINATES (FIXED)

DIMENSION Y(1),Z(1)

DO 1 I=1,NO

YS = Y(I)

Y(I) = Z(I)

1 Z(I) = -YS

RETURN

END

SUBROUTINE TRANS3 (Y,Z,V,W,NO,KS,NS,TNEG,NLST)

TRANSFORMS PROGRAM COORDINATES (FIXED) TO OUTPUT COORDINATES,  
JET CENTERLINE AND CONTROL POINT COORDINATES ARE AFFECTED

DIMENSION X1(11,10),Z1(11,10),UJ1(11,10),D1(11,10),DXDZ1(11,10)  
DIMENSION X2(11,10),Z2(11,10),UJ2(11,10),D2(11,10),DXDZ2(11,10)

```

DIMENSION X1T(100),Z1T(100),UJ1T(100),D1T(100),DXDZ1T(100)
DIMENSION X2T(100),Z2T(100),UJ2T(100),D2T(100),DXDZ2T(100)
DIMENSION XBAS1(100),YBAS1(100),ZBAS1(100)
DIMENSION XBAS2(100),YBAS2(100),ZBAS2(100)
DIMENSION XBAS3(100),YBAS3(100),ZBAS3(100)
DIMENSION XJ1(10),YJ1(10),ZJ1(10),DJET1(10),VELJ1(10)
DIMENSION XJ2(10),YJ2(10),ZJ2(10),DJET2(10),VELJ2(10)

```

C

```

COMMON/BLK2/X1,Z1,UJ1,D1,DXDZ1,X2,Z2,UJ2,D2,DXDZ2
COMMON/BLK3/X1T,Z1T,UJ1T,D1T,DXDZ1T,X2T,Z2T,UJ2T,D2T,DXDZ2T
COMMON/BLK5/XBAS1,YBAS1,ZBAS1,XBAS2,YBAS2,ZBAS2,XBAS3,YBAS3,ZBAS3
COMMON/BLK9/MULT,IHOLD1,KOUNT1,IONE,ITWO,ITHR,N1,N2,N3,IFIX1
COMMON/BLK10/XJ1,YJ1,ZJ1,DJET1,VELJ1,XJ2,YJ2,ZJ2,DJET2,VELJ2

```

C

```

DIMENSION Y(1),Z(1),V(1),W(1)

```

C

```

DO 1 I=1,NO
YS = Y(I)
Y(I) = -Z(I)
Z(I) = YS
VS = V(I)
V(I) = -W(I)
1 W(I) = VS
KTR1 = 0
KTR2 = 0
IF (MULT-2) 8,5,5
5 IF (TNEG) 6,6,7
6 KTR2 = KOUNT1
GO TO 8
7 KTR1 = KOUNT1
8 CONTINUE
UJ1T(1) = UJ1(1,1)
D1T(1) = D1(1,1)
JK = 1
DO 20 I=1,KS
IF (I-1) 10,10,11
10 NF = NS+1-KTR1
GO TO 15
11 IF (I-KS) 12,13,13
12 NF = NS+1
GO TO 15
13 NF = NLST
15 CONTINUE
DO 20 J=2,NF
JK = JK+1
UJ1T(JK) = UJ1(J,I)*VELJ1(I)/VELJ1(1)
20 D1T(JK) = D1(J,I)*DJET1(I)/DJET1(1)
IF (MULT-2) 50,25,25
25 UJ2T(1) = UJ2(1,1)
D2T(1) = D2(1,1)
JK = 1
DO 40 I=1,KS
IF (I-1) 30,30,31
30 NF = NS+1-KTR2
GO TO 35

```

```

31 IF (I-KS) 32,33,33
32 NF = NS+1
   GO TO 35
33 NF = NLST
35 CONTINUE
   DO 40 J=2,NF
     JK = JK+1
     UJ2T(JK) = UJ2(J,I)*VELJ2(I)/VELJ2(1)
40  D2T(JK) = D2(J,I)*DJET2(I)/DJET2(1)
50 CONTINUE
   DO 52 I=1,N1
     YS = YBAS1(I)
     YBAS1(I) = -ZBAS1(I)
52  ZBAS1(I) = YS
     IF (N2) 60,60,53
53  DO 54 I=1,N2
     YS = YBAS2(I)
     YBAS2(I) = -ZBAS2(I)
54  ZBAS2(I) = YS
     IF (N3) 60,60,55
55  DO 56 I=1,N3
     YS = YBAS3(I)
     YBAS3(I) = -ZBAS3(I)
56  ZBAS3(I) = YS
60 CONTINUE
   RETURN
   END

```

SUBROUTINE OUTPT (XL,ZL,DXDZ,CF,DJ,XJ,YJ,ZJ,XB,YB,ZB,VX,VY,VZ)

```

C
C TRANSFORMS LOCAL COORDINATES TO PROGRAM COORDINATES (FIXED)
C
C DIMENSION CF(3,3)
C
C PHI = ATAN(DXDZ)
  VXT = SIN(PHI)
  VYT = 0.
  VZT = COS(PHI)
  CALL ROTATE (VX,VY,VZ,CF,VXT,VYT,VZT,1)
  CALL ROTATE (FX,FY,FZ,CF,XL,0.,ZL,1)
  XB = FX*DJ+XJ
  YB = FY*DJ+YJ
  ZB = FZ*DJ+ZJ
  RETURN
  END

```

SUBROUTINE OUTPT1 (XL,ZL,DXDZ,CF,KS,DJ,XJ,YJ,ZJ,XB,YB,ZB,  
1 VX,VY,VZ)

```

C
C TRANSFORMS LOCAL COORDINATES TO PROGRAM COORDINATES (FIXED),FOR
C THE SEGMENTED JETS
C
C DIMENSION CF(3,3,10)

```

```

PHI = ATAN(DXDZ)
VXT = SIN(PHI)
VYT = 0.
VZT = COS(PHI)
CALL ROTATE (VX,VY,VZ,CF(1,1,KS),VXT,VYT,VZT,1)
CALL ROTATE (FX,FY,FZ,CF(1,1,KS),XL,0.,ZL,1)
XB = FX*DJ+XJ
YB = FY*DJ+YJ
ZB = FZ*DJ+ZJ
RETURN
END

```

```

SUBROUTINE PLANE (CFN1,CFN2,CFN3,X1,Y1,Z1,CSN1,CSN2,CSN3,XL1,XL2,
1 XL3,COOR1,COOR2,COOR3)

```

```

C COMPUTES INTERSECTION OF A GIVEN PLANE WITH A LINE
C
C

```

```

C DIMENSION CFN(3),CSN(3),XL(3),COOR(3)

```

```

CFN(1) = CFN1
CFN(2) = CFN2
CFN(3) = CFN3
CSN(1) = CSN1
CSN(2) = CSN2
CSN(3) = CSN3
XL(1) = XL1
XL(2) = XL2
XL(3) = XL3

```

```

IL = 1

```

```

IM = 1

```

```

IN = 1

```

```

SUB1 = 0.

```

```

IF (ABS(CSN(1))-1.0E-04) 1,1,2

```

```

1 IL = 0

```

```

SUB1 = CFN(1)*XL(1)

```

```

COOR(1) = XL(1)

```

```

2 IF (ABS(CSN(2))-1.0E-04) 3,3,4

```

```

3 IM = 0

```

```

SUB1 = SUB1+CFN(2)*XL(2)

```

```

COOR(2) = XL(2)

```

```

4 IF (ABS(CSN(3))-1.0E-04) 5,5,6

```

```

5 IN = 0

```

```

SUB1 = SUB1+CFN(3)*XL(3)

```

```

COOR(3) = XL(3)

```

```

6 D = CFN(1)*X1+CFN(2)*Y1+CFN(3)*Z1

```

```

IF (IL+IM+IN-2) 10,30,50

```

```

10 IF (IL) 12,11,12

```

```

11 IF (IM) 14,13,14

```

```

12 IP = 1

```

```

GO TO 15

```

```

14 IP = 2

```

```

GO TO 15

```

```

13 IP = 3

```

```

15 COOR(IP) = (D-SUB1)/CFN(IP)
   GO TO 90
30 IF (IL) 32,31,32
31 IP1 = 2
   IP2 = 3
   GO TO 35
32 IF (IM) 34,33,34
33 IP1 = 1
   IP2 = 3
   GO TO 35
34 IP1 = 1
   IP2 = 2
35 SLOPE = CSN(IP1)/CSN(IP2)
   COOR(IP2) = (D-SUB1+CFN(IP1)*SLOPE*XL(IP2)-CFN(IP1)*XL(IP1))/
1       (CFN(IP1)*SLOPE+CFN(IP2))
   COOR(IP1) = SLOPE*(COOR(IP2)-XL(IP2))+XL(IP1)
   GO TO 90
50 COEFX1 = 1./CSN(1)
   COEFY1 = -1./CSN(2)
   D1 = XL(1)/CSN(1)-XL(2)/CSN(2)
   COEFX2 = 1./CSN(1)
   COEFZ2 = -1./CSN(3)
   D2 = XL(1)/CSN(1)-XL(3)/CSN(3)
   CALL SOL (CFN(1),CFN(2),CFN(3),D,COEFX1,COEFY1,0.,D1,COEFX2,0.,
1 COEFZ2,D2,COOR(1),CCOR(2),COOR(3))
90 COOR1 = COOR(1)
   COOR2 = COOR(2)
   COOR3 = COOR(3)
   RETURN
   END

```

```

SUBROUTINE ADAMS(N,START,FINAL,H,PRINT,ICOUNT,RELB,ABSB,ISKIP,
1 XO,XP,PAR,DDERIV)

```

```

C
C SUBROUTINE ADAMS SOLVES A SYSTEM OF *N* FIRST ORDER DIFFERENTIAL
C EQUATIONS BY MEANS OF A FOURTH ORDER ADAMS PREDICTOR/CORRECTOR
C METHOD. THE STARTING SOLUTION IS BY RUNGE-KUTTA METHOD.
C AUTOMATIC ERROR CONTROL IS OPTIONAL.
C
C DIMENSION X(50,5),VK(50,4),F(50,5),E(50)
C DIMENSION XP(1),XO(1),PAR(1)
C
C IBCOL = 0
C IF (PRINT) 20,10,20
10 IF (ICOUNT) 20,31,20
C
20 CONTINUE
C20 WRITE (6,400) ID,N
   IBCOL = 1
C400 FORMAT (17HOPROBLEM NUMBER I10,5X12HSOLUTION OF
1 I3,5X35HFIRST ORDER DIFFERENTIAL EQUATIONS.)
C
C SETUP INITIAL VALUES

```

```

DO 30 I=1,N
X(I,1) = X0(I)
30 CONTINUE
31 CONTINUE
IF (ICOUNT) 40,35,40
35 ICOUNT = 9999
40 ITEMP = 0
BOUND = START+PRINT
T = START
IF (ISKIP) 45,50,45
45 IA = 2
IB = 4
GO TO 2222
50 RLTEST = 14.2*RELB
ABTEST = 14.2*ABS8
FACTOR = RELB/ABS8
BLB = RLTEST/200.0
H = 2.0*H
C
C RUNGE-KUTTA STARTING METHOD
C
1111 IA = 2
IB = 2
C
2222 DO 90 J=IA,IB
CALL DDERIV (T,X(1,J-1),F(1,J-1),PAR)
DO 60 I=1,N
VK(I,1) = H*F(I,J-1)
X(I,J) = X(I,J-1)+.5*VK(I,1)
60 CONTINUE
TTEMP = T+.5*H
C
CALL DDERIV (TTEMP,X(1,J),F(1,J),PAR)
DO 70 I=1,N
VK(I,2) = H*F(I,J)
X(I,J) = X(I,J-1)+.5*VK(I,2)
70 CONTINUE
C
CALL DDERIV (TTEMP,X(1,J),F(1,J),PAR)
DO 80 I=1,N
VK(I,3) = H*F(I,J)
X(I,J) = X(I,J-1)+VK(I,3)
80 CONTINUE
T = T+H
C
CALL DDERIV (T,X(1,J),F(1,J),PAR)
DO 85 I=1,N
VK(I,4) = H*F(I,J)
X(I,J) = X(I,J-1)+.16666667*(VK(I,1)+2.0*(VK(I,2)+
1 VK(I,3))+VK(I,4))
85 CONTINUE
90 CONTINUE

```

```

      IF (IB-2) 150,3333,150
3333 DO 100 I=1,N
      XP(I) = X(I,2)
100  CONTINUE
C
C      XP(I)=DOUBLE INTERVAL RESULT TO BE USED IN ERROR
C      ANALYSIS
C
      T = T-H
      H = .5*H
C
      IF (IB00L) 120,125,120
120  CONTINUE
C120 WRITE (6,410) H
C410 FORMAT (34H0IN THE FOLLOWING CALCULATIONS H =E14.8)
125  IF (H-.0000001) 130,130,140
130  WRITE (6,420)
420  FORMAT (1H0,10(1H*),////
1      49HOEQUATIONS CAN NOT BE SOLVED FURTHER WITHIN GIVEN
2      14H ERROR BOUNDS.)
      RETURN
C
140  IB = 3
      GO TO 2222
C
150  IF (IB-3) 200,160,200
C
C      IS ACCURACY CRITERION MET
C
160  J = 3
4444 DO 190 I=1,N
      E(I)=ABS(XP(I)-X(I,J))
      IF(E(I)-ABS(X(I,J)*RLTEST))170,175,175
170  E(I)=E(I)/ABS(X(I,J))
      GO TO 190
175  IF (E(I)-ABTEST) 180,185,185
180  E(I) = E(I)*FACTOR
      GO TO 190
C
185  T =T-H
      IF (J-5) 3333,187,3333
187  DO 188 K=1,N
188  X(K,1) = X(K,4)
      GO TO 1111
190  CONTINUE
C
      IF (J-5)195,6666,195
195  IA = 4
      IB = 4
      GO TO 2222
C
C      SHOULD ANY OF THE STARTING VALUES BE PRINTED OUT
C
200  T = T-3.0*H

```



```

DO 250 J=2,4
T = T+H
ITEMP = ITEMP+1
IF (PRINT) 210,230,210
210 IF (T-BOUND) 230,220,220
220 BOUND = BOUND+PRINT
9999 CONTINUE
C9999 WRITE (6,430) T,(I,X(I,J),I=1,N)
C430 FORMAT (4H0T =E14.8/ 5( 2H X,I2,1H=1PE12.5))
ITEMP = 0
C
230 IF (ITEMP-ICOUNT) 240,9999,240
240 IF (T-(FINAL-H/10.0)) 250,999,999
250 CONTINUE
C
C BEGIN ADAMS METHOD
C
5555 CALL DDERIV (T,X(1,4),F(1,4),PAR)
DO 260 I=1,N
XP(I) = X(I,4)+.041666667*H*(55.0*F(I,4)-59.0*F(I,3)
1 +37.0*F(I,2)-9.0*F(I,1))
260 CONTINUE
C
T = T+H
CALL DDERIV (T,XP(1),F(1,5),PAR)
DO 270 I=1,N
X(I,5) = X(I,4)+.041666667*H*(9.0*F(I,5)+19.0*F(I,4)-
1 5.0*F(I,3)+F(I,2))
270 CONTINUE
C
IF (ISKIP) 6666,280,6666
280 J = 5
GO TO 4444
C
6666 IF (T-(FINAL-H/10.0)) 295,290,290
290 J = 5
GO TO 999
C
295 DO 300 I=1,N
X(I,4) = X(I,5)
DO 300 J=2,5
F(I,J-1) = F(I,J)
300 CONTINUE
C
ITEMP = ITEMP+1
C
C TEST WHETHER COMPUTED VALUES SHOULD BE PRINTED
C
IF (PRINT) 310,330,310
310 IF (T-(BOUND-H/10.0))330,320,320
320 BOUND = BOUND+PRINT
7777 J = 4
C WRITE (6,430) T,(I,X(I,J),I=1,N)

```

```

      ITEM = 0
C
330 IF (ITEM-ICOUNT) 340,7777,340
340 IF (ISKIP) 5555,350,5555
C
C   TEST WHETHER INTERVAL CAN BE DOUBLED
C
350 DO 355 I=1,N
      IF (E(I)-BLB) 355,355,5555
355 CONTINUE
C
      IF (PRINT) 358,380,358
358 D1 = PRINT/(2.0*H)
      D1I=ABS(FLOAT(IFIX(D1))-D1)
      IF (D1I-.1) 362,362,360
360 IF (D1I-.9) 5555,362,362
362 D2 = (BOUND-T)/(2.0*H)
      D2I=ABS(FLOAT(IFIX(D2))-D2)
      IF (D2I-.1) 380,380,365
365 IF (D2I-.9) 5555,380,380
380 DO 382 I=1,N
      X(I,1) = X(I,4)
382 CONTINUE
      H = 4.0*H
      GO TO 1111
C
999 CONTINUE
C999 WRITE (6,440)
C440 FORMAT (20HOFINAL T AND XP()...)
      DO 385 I=1,N
      XP(I) = X(I,J)
385 CONTINUE
      FINAL = T
C   WRITE (6,430) T,(I,X(I,J),I=1,N)
      RETURN
      END

```

# SUBROUTINE CFCAL (ALFQ,BETQ,GETQ,CXJ,CYJ,CZJ,CF)

```

C
C   COMPUTES DIRECTION COSINES FOR THE LOCAL COORDINATE SYSTEM, X IN
C   DIRECTION OF FREESTREAM,Y NORMAL TO FREESTREAM AND INITIAL JET
C   DIRECTION, Z IS XCROSSY
C
C   DIMENSION CF(3,3)
C
      CF(1,1) = ALFQ
      CF(1,2) = BETQ
      CF(1,3) = GETQ
      CALL XPROD (CXJ,CYJ,CZJ,CF(1,1),CF(1,2),CF(1,3),CF(2,1),CF(2,2),
1 CF(2,3))
      CALL XPROD (CF(1,1),CF(1,2),CF(1,3),CF(2,1),CF(2,2),CF(2,3),
1 CF(3,1),CF(3,2),CF(3,3))
      RETURN
      END

```

SUBROUTINE CFCAL1 (ALF,BET,GET,CXJ,CYJ,CZJ,CF,K)

GIVES DIRECTION COSINES FOR THE JET-ORIENTED COORDINATE SYSTEM  
X-AXIS IS IN DIRECTION OF FREESTREAM,Y IS NORMAL TO THE PLANE  
DEFINED BY THE FREESTREAM AND JET EXHAUST DIRECTIONS,Z AXIS IS  
X-CROSS-Y. CFCAL1 SAME AS CFCAL EXCEPT FOR PARAMETER K

DIMENSION CF(3,3,10)

CF(1,1,K) = ALF

CF(1,2,K) = BET

CF(1,3,K) = GET

CALL XPROD (CXJ,CYJ,CZJ,CF(1,1,K),CF(1,2,K),CF(1,3,K),CF(2,1,K),  
1 CF(2,2,K),CF(2,3,K))

CALL XPROD (CF(1,1,K),CF(1,2,K),CF(1,3,K),CF(2,1,K),CF(2,2,K),  
1 CF(2,3,K),CF(3,1,K),CF(3,2,K),CF(3,3,K))

RETURN

END

SUBROUTINE ROTATE (A,B,C,CF,S,T,U,L)

L=0 ROTATES A,B,C INTO S,T,U,(FIXED COORDINATES TO ROTATED)

L=1 ROTATES S,T,U INTO A,B,C,(ROTATED COORDINATES TO FIXED)

DIMENSION CF(3,3),D(3),V(3)

IF (L) 1,1,2

1 D(1) = A

D(2) = B

D(3) = C

GO TO 3

2 D(1) = S

D(2) = T

D(3) = U

3 CONTINUE

DO 4 I=1,3

4 V(I) = 0.

DO 5 I=1,3

DO 5 J=1,3

IF (L) 9,9,10

9 M = I

N = J

GO TO 5

10 M = J

N = I

5 V(I) = V(I)+D(J)\*CF(M,N)

IF (L) 6,6,7

6 S = V(1)

T = V(2)

U = V(3)

GO TO 8

```

7  A = V(1)
   B = V(2)
   C = V(3)
8  CONTINUE
   RETURN
   END

```

SUBROUTINE XPROD (ALF1,BET1,GET1,ALF2,BET2,GET2,ALF3,BET3,GET3)

C  
C  
C COMPUTES CROSS PRODUCT OF TWO VECTORS, RETURNS A UNIT VECTOR

```

ALF3 = BET1*GET2-BET2*GET1
BET3 = ALF2*GET1-ALF1*GET2
GET3 = ALF1*BET2-ALF2*BET1
DENOM = SQRT(ALF3*ALF3+BET3*BET3+GET3*GET3)
ALF3 = ALF3/DENOM
BET3 = BET3/DENOM
GET3 = GET3/DENOM
RETURN
END

```

SUBROUTINE SOL (A11,A12,A13,AK1,A21,A22,A23,AK2,A31,A32,A33,AK3,  
1 X1,X2,X3)

C  
C  
C SOLVES A SET OF THREE EQUATIONS BY METHOD OF DETERMINANTS

```

DELTA = A11*(A22*A33-A23*A32)+A21*(A32*A13-A12*A33)
1      +A31*(A12*A23-A13*A22)
X1 = (AK1*(A22*A33-A23*A32)+AK2*(A32*A13-A12*A33)
1      +AK3*(A12*A23-A13*A22))/DELTA
X2 = (A11*(AK2*A33-A23*AK3)+A21*(AK3*A13-AK1*A33)
1      +A31*(AK1*A23-A13*AK2))/DELTA
X3 = (A11*(A22*AK3-AK2*A32)+A21*(A32*AK1-A12*AK3)
1      +A31*(A12*AK2-AK1*A22))/DELTA
RETURN
END

```